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An iPhone App to Demonstrate Locational Marginal Pricing in Electricity Markets

STOR 489 Project (30 points)

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Abstract

This project is concerned with Locational Marginal Pricing (LMP), the methodology used to determine wholesale electricity prices in New Zealand. LMP is also used by electricity markets in Australia, Singapore and half of the electricity markets in the USA. Despite its widespread use, it is difficult to find an explanation of LMP that does not require specialist knowledge and it is even more difficult to gain access to a system the will allow for experimentation with a functioning LMP model.

This report explains the background necessary to understand the LMP formulation then presents the software that was developed as the the basis for this project; a standalone LMP system that runs on an iPhone or iPad. This "LMP app" allows an LMP model to be built, solved and analysed on the device, with the option of exporting the results for more detailed analysis and reporting. Naturally the size of the device limits the size of the model, but it is shown that the LMP app can be used to produce models that demonstrate all significant features of LMP. It is also shown how the LMP app can be used to model sections of the actual New Zealand electricity market system, and to illustrate proposed electricity market changes.

Contents

1 Introduction

1.1 Electricity Markets and Locational Marginal Pricing (LMP)

LMP uses the Operations Research method of Linear Programming to decide which generators will meet the demand for electricity, while the dual of this result determines the electricity prices. New Zealand was the first country to implement LMP, in 1996 [60]. The United States is divided into ten electricity markets; five of these now use LMP [16]. LMP is also used in Singapore, Australia and other countries [62]. Electricity markets in Europe and China are investigating LMP [41][78]. Where did LMP come from and what makes it important?

Worldwide, the deregulation of the supply of electricity began in the 1980s. Prior to this the cost of electricity everywhere was subject to some form of state regulation. In many countries it still is, but most developed countries have now implemented deregulation in the form of an electricity market where the wholesale electricity price is determined by market forces [13].

In the New Zealand's electricity market (as at 2013), generating companies (e.g. Genesis Energy) register to act as *generators* of electricity at a specified location (e.g. Huntly Power Station). Electricity retailers (e.g. Mercury Energy) register to act as *purchasers* of electricity at a specified location (e.g. Hamilton). The wholesale electricity market determines how much the generator is paid and how much the purchaser pays. It is referred to as the *wholesale* electricity market because the purchaser buys electricity in bulk and then on-sells to the end consumer[1]. For brevity we will drop the word wholesale from now on and simply refer to the *electricity market*.

Under LMP, the electricity price is *locational*; the price can be different at different geographical locations because the LMP model incorporates a representation of the electrical network that transports electricity from generators to loads. The model includes constraints which represent the physical laws that apply to the flow of electricity; these laws determine which path the electricity takes, what limits are placed on each path and the level of losses that are incurred. All of these factors contribute to the cost of providing the electricity to a specific location.

The locational price represents the *marginal* benefit of providing another unit of electricity to that location, regardless of the cost for lesser quantities. For example, if it cost $100 per unit to provide a total of 49 units to a location and $200 per unit to provide anything more than that, then if the LMP solution produced a result of 50 units then the price would be $200 per unit for all 50 units.

Sufficient electricity must be generated to meet the *real time* load, 24 hours a day, seven days a week. The matching of generation to load is the role of the *System Operator*. The System Operator *dispatches* a generation quantity to each of the generators. The *dispatch* quantity is determined by the primal solution of the LMP model. The System Operator is

[1] In 1989, when the author was working for the Taranaki Electric Power Board (TEPB), electricity users had no choice who they bought their electricity from; if you lived in Taranaki you had to buy electricity from the TEPB. However, TEPB was making preparations for the deregulation that would eventually see the New Zealand electricity market starting in 1996. As part of this preparation, one of the things we were told to do was to stop referring to electricity purchasers as consumers and start referring to them as customers, because consumer implied that they had no choice, whereas customer implied the ability to shop around, even though at that time such a choice was still many years away. In this report I use the term consumer because it is less ambiguous.

solving the model with real time load inputs every five minutes.

The dual of the LMP model provides the wholesale electricity prices. These are the electricity prices that apply to countries, e.g., New Zealand, Singapore, or in the case of the USA, large regions of a country [16]. Generating companies use these prices when they decide whether or not to invest in building new generation [20]. The electricity prices paid by consumers has the potential to impact on the economy of the country. The Smithsonian Institute has a section of their website where they document electricity restructuring. In 1998 after the first electricity market in the USA implemented LMP the Smithsonian wrote: "If LMP becomes an industry standard, then it would play a role in determining the price you pay for electricity in a deregulated market" [34]. LMP is now the industry standard in the USA [16]. The LMP model produces results that have a financial impact. In order for there to be confidence in the electricity market there are times when these results need to be explained. When changes to the electricity market are proposed, in order for informed debate to occur it would be useful if these proposals could be illustrated by interactive demonstration.

1.2 Existing demonstrations of LMP

There is no easy way to find out more about the LMP model that produces these important results. There are some papers [43][40][26] that discuss LMP at the level of detailed formulation, but these assume an already detailed knowledge of the electrical engineering theory or the mathematical theory underlying the LMP model. There are also some presentations that explain LMP at a more basic level [54][65][38]. These assume a level of knowledge about the system being modeled and, while they explain the outcomes of the formulation, the formulation itself is not investigated in detail. They show what happens, without answering all the questions about why. Models are presented that demonstrate the effects of LMP, but what is missing is the ability to interact with these models; to see what would happen if the configuration was slightly different, or the input values were changed; to confirm by investigation, to learn by doing.

The author was a scheduler for the New Zealand electricity market when it started in October 1996 and subsequently moved into an Information Technology (IT) role to provide support for the electricity market software. From 2002 to 2006 he was the LMP model specialist for the Singapore electricity market, which went live in January 2003. Since 2006 his work has involved supporting the LMP software used by the New Zealand electricity market. In these roles there are occasions when it has been necessary to explain LMP. A technique that the author developed which allowed for visualization of the model, combined with the ability to easily modify some of the inputs, was to use a customised Excel spreadsheet as shown in Figure 1. Excel's drawing tools can be used to create a graphical representation of the system being modeled, with the associated parameters entered into the adjacent cells. The constraints of the LMP model are created using Excel's Solver add-in [77], which is then used to solve the LP. The results are written to an adjacent worksheet and, with some background code and a few lookup formulas, the prices, quantities and flow arrows can be displayed next to the associated components on the graphical model. The limitations of this approach are that individually selecting cells to build the constraints can be time consuming even for a simple model and changing the model requires re-drawing, re-defining cell names and careful modification of the constraints. Also, there is no way to view the internal workings of the Solver.

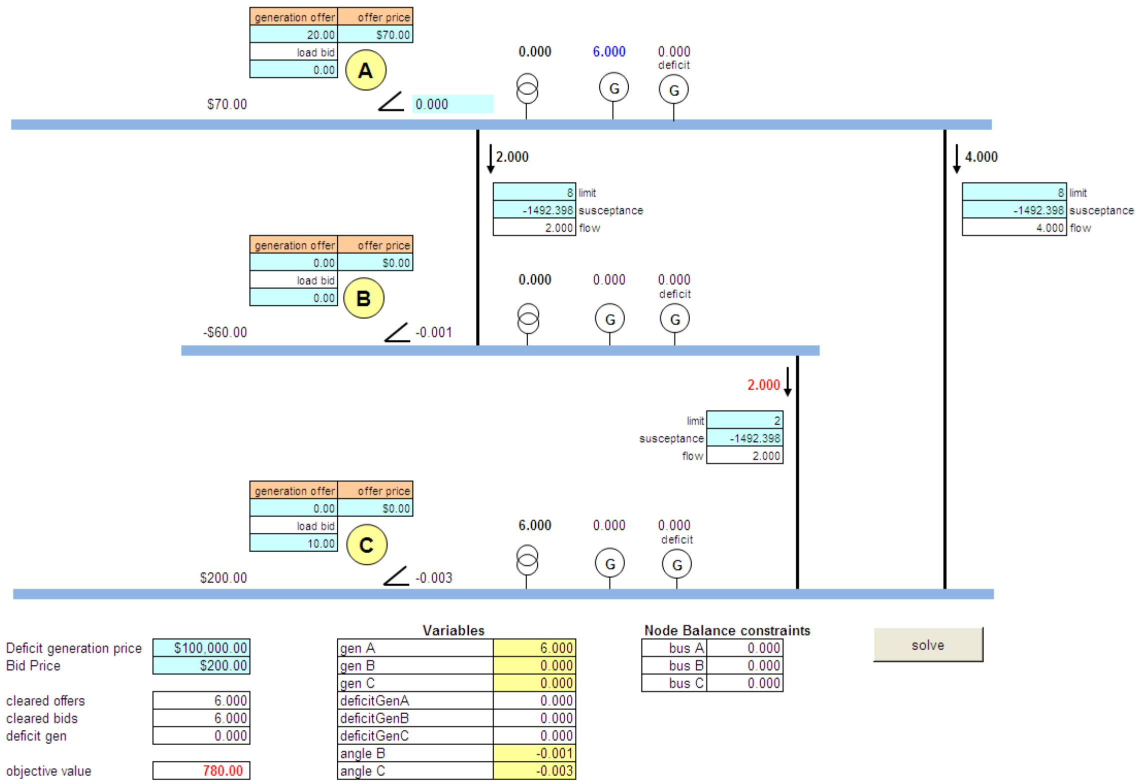


Figure 1: An LMP model built in Excel.

The Harvard Electricity Policy Group offers "toy models" to explain LMP [30] but these are non-graphical and also require access to the GAMS solver, which is several hundred dollars for a student licence and several thousand for a commercial licence [19]. The New Zealand Electricity Authority also offers an LMP model for free download [7]. However, the intent of this model is to replicate the results produced by the New Zealand electricity market; it is not graphical, the workings of the code are hidden and the GAMS solver is required.

Ideally an application to *demonstrate* LMP would include the ability to quickly and easily build or change the system being modeled, as well as allowing for pre-prepared cases to be loaded and experimented with. It would also be good if the software was cheap, portable and easy to install.

1.3 Goal of this project

The workings of an LMP electricity market can appear to be quite complicated. This project explores the idea that it is possible to break down the concepts of the LMP model to a level where they can be explained, reasonably concisely, to someone with no background in electricity markets or power systems. This explanation is combined with an iPhone app that was created as part of this project, which allows the user of the app to build and solve an LMP model. The app is shown in Figure 2 and hereafter referred to as the *LMP app*.

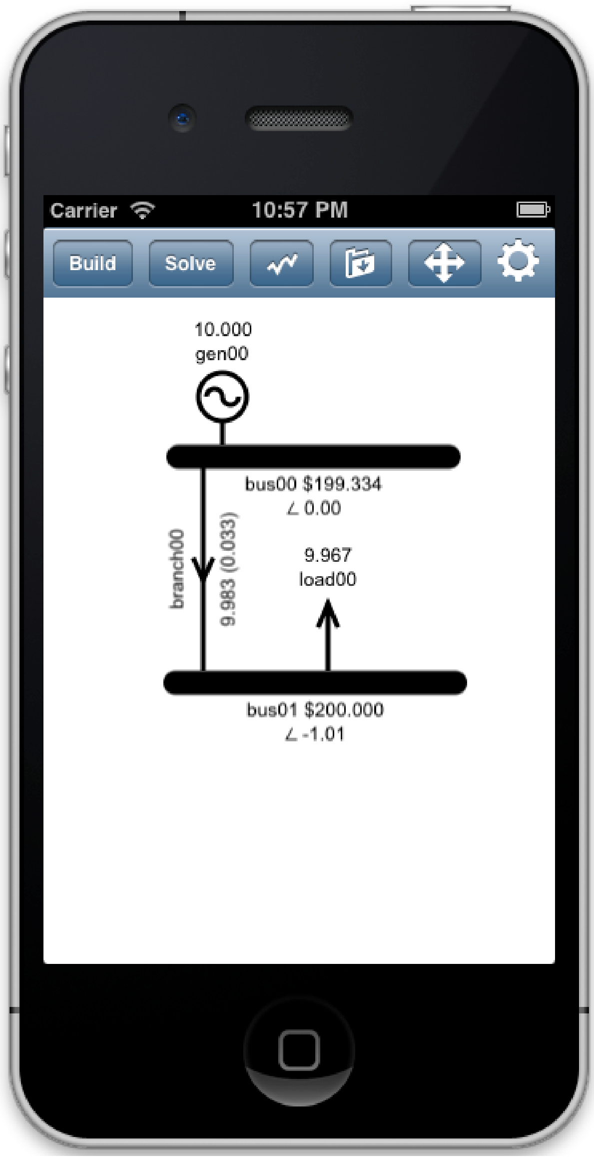


Figure 2: The LMP app.

Specific questions addressed by this project include the following. Are there enough things that can be done with the LMP app to make it worthwhile? How well does the LMP app demonstrate the basics and the more advanced scenarios? Is it capable of modelling real world scenarios? How well does a result produced by the LMP app compare to an LMP model implemented by another system?

1.4 Outline of this report

In Section 5 we will use the LMP app to demonstrate how LMP works and explain some of the more interesting results that LMP can produce. Also we want to show how the LMP app can be used to model small versions of real world scenarios. The overarching goal is that these explanations and demonstrations will make sense to a person who has never heard of LMP before picking up this report. Hence, sections leading up to Section 5 provide all the necessary background for that person.

LMP operates within the electricity market, which is a relatively recent introduction; Section 2 places the electricity market in the context of the history of electricity supply. We start from the beginnings in the 1880's when electricity supply was a deregulated industry and describe how it moved to became a regulated industry for many years, before the deregulation that began in the 1980's which led to electricity markets and LMP.

Section 3 presents the the power system, in detail. The LMP formulation models the physical system, the power system, that provides electricity to homes and factories. A comprehensive understanding of the "what", "why" and "how" of the power system is necessary in order to make sense of the LMP formulation. As well as describing what the power system is, we also explain why the power system is constructed in the way that it is. This requires some basic electrical theory (electrical power and electrical energy; Ohms Law) which is explained with examples. We also describe how the power system is managed by the System Operator; it is necessary to describe how the power system is managed, because this also influences the LMP formulation.

2 Background History of Electricity Markets

In order to explain electricity markets, we will start with a brief history of electricity supply. After that we will describe how electricity supply came to be a regulated industry for much of the twentieth century, before heading towards de-regulation from the 1980's. This will be followed by a brief history of de-regulated electricity markets that implemented LMP, ending with an overview of the functionality of SPD, the New Zealand electricity market software.

2.1 A brief history of Electricity Supply

The first investor-owned electricity utility was opened by Thomas Edison in Pearl Street, New York in September 1882. This system provided direct current (DC) electricity. There were issues with transmitting DC electricity over long distances. This was because DC voltage cannot be easily changed; the transmission voltage is the voltage received into the home, hence the transmission voltage in the DC system could not be too high or the voltage in the home would be dangerous. Low transmission voltages lead to relatively high losses, and this restricted the distance between generation and load to no more than several miles [33].

In 1885, engineers working for George Westinghouse perfected a transformer which allowed the voltage of alternating current (AC) electricity to be changed. This paved the way for a commercial AC electricity system to be introduced in 1886. This system was in direct competition with the Edison system because the equipment that produced, transmitted and consumed the AC electricity was not compatible with DC equipment. By the 1890's the AC system prevailed; because the AC electricity could be transmitted at high voltages with low losses and then changed to a lower voltage to make it safe to use, it allowed electricity to be transmitted across long distances [33]. This in turn allowed generation opportunities to be explored that were not close to the load, the first of which was hydroelectric generation built by Westinghouse at Niagra Falls which went live in 1896 [42]. The following years saw small scale commercial distribution of electricity spread across the USA, England and Europe [13].

In the USA, the UK and parts of Europe, for its first twenty years the electricity industry was unregulated and commercial [64]. In other places the government controlled electricity production and supply from the beginning, e.g., the Water Act of 1903 in New Zealand gave the Crown the right to use water for hydroelectric generation [5]

From the 1920s onwards things changed in those places were electricity was unregulated; the state increasingly became a regulator or producer of electricity as it became obvious that electricity supply was vital to the economic development of the country. In the USA there were attempts to regulate at the state level, but this proved difficult because electricity supply crossed state boundaries. In 1935, Congress passed the Public Utility Holding Company Act which placed restrictions on what electricity suppliers could own and allowed for control over prices [70]. Following World War Two, electricity usage experienced strong growth. The UK electricity supply industry was nationalised in 1947 [13]. Developing countries building electricity systems with the assistance of the cold war Super Powers followed the same regulated path [51].

In the late 1960s growth in electricity usage started to slow and this was compounded by the oil crisis of the 1970s. The regulated electricity industry suffered from the fact that generation planning had been based on the assumption that growth would continue at a rapid rate; slowing growth resulted in an over supply of generation which had been built with little regard to cost. Increases in the regulated price were required to pay for this expensive generation, despite the fact that demand was dropping, and these price rises were not distributed evenly. The problems were highlighted by the fact that new small scale generation was becoming available in the form of natural gas, but it was difficult for this generation to enter the regulated system [64][70].

One option was to regulate a way out of these problems. However, from an economists point of view, another solution was to let a market solve these problems; the market would take care of electricity production, while the government role would be to promote competition and ensure fairness. In response to the oil crises, the USA passed the 1978 Public Utilities Regulatory Policies Act (PURPA), which included the aim of moving electricity supply toward market-based prices [64].

However, taking concrete action was politically difficult. The first country to comprehensively reform its electricity industry was Chile, where the reforms began in 1982 and involved separation of the entities responsible for generating electricity from those responsible for its transmission and distribution. This was followed by large-scale privatisation of the generation. This was done with little fear of political repercussions as it was put in place by the military government of Augusto Pinochet [56].

Over the next two decades other countries followed a similar process of separating the generation assets (the energy), which represent the portion of the industry which is theoretically contestable, from the transmission and distribution assets (the lines) where there is no point in multiple companies building separate wires to the same town or the same house.

The next country to take concrete action on progressing electricity reform was the UK under the conservative government of Margaret Thatcher. The Electricity Act of 1983 promoted similar aims to PURPA, and was followed by legislation in 1989 which separated lines and energy, with the compulsory electricity pool opened in 1990. Although it was subsequently reformed, this was the first electricity market [13].

In 1986 the New Zealand government signaled its intentions to reform the electricity supply industry and in 1987 the Electricity Corporation of New Zealand (ECNZ) was created as a State Owned Enterprise (SOE) responsible for generation and transmission, separate from ministerial control [52]. In 1988, Transpower was set up as a subsidiary of ECNZ to manage the transmission assets, leaving ECNZ with generation only. In July 1994 Transpower became an SOE separate from ECNZ. The author joined Transpower in January 1994, working for the System Operator division. In its role as the System Operator, Transpower was responsible for securely running the power system, which included scheduling and dispatching generation [2]. In October 1996, New Zealand implemented the first electricity

[2]Prior to the start of the electricity market in New Zealand, the System Operator scheduled generation based on prices provided by ECNZ; each hydro chain (set of hydroelectric power stations using water from the same source) were assigned a water price, e.g., the group of generators on the Waikato river (i.e. the group of generators now referred to as Mighty River Power) were scheduled based on the Taupo water price assigned by ECNZ. Thermal generators were assigned a price based on the cost of gas and the efficiency of the generator. In those days, Huntly power station was a relatively efficient thermal generator while New Plymouth power station was relatively inefficient. New Plymouth power station has since been de-commissioned. Newer thermal units have been built which use modern combined cycle gas turbine (CCGT) technology and Huntly power station is now relatively inefficient, although there is no longer a central authority making such decisions. Huntly power station is in the process of de-commissioning units.

market where scheduling, pricing and dispatch were determined by linear programming, locational marginal pricing model. No longer did ECNZ provide prices for water usage and running costs for thermal units; every generator offered their generation at a price, with up to five price-quantity bands.

2.2 The proposal for an electricity spot market

LMP traces its beginnings back to 1982 when Caramanis, Bohn and Schweppe from the Energy Laboratory at the Massachusetts Institute of Technology proposed the theoretical basis for an electricity spot market [14]. Under this proposal, electricity prices would be calculated based on the state of the power system at any instant in time. The calculation model would include constraints to simulate the power system that delivers the generation to the load. Each metered point on the system would have its own price, and this price would be the same whether buying or selling electricity.

The system would be driven by social welfare and cost functions, where social welfare was the benefit resulting from supplying load, e.g., supplying electricity to a factory or a house, and cost was what resulted from running a generator or the financial losses incurred due to enforced load reduction occurring due to electricity shortages. The objective would be to maximise social welfare and minimise cost. Maximising this objective would determine which generators would generate and also set the electricity spot market price.

The price of electricity at a location would reflect the value of taking or supplying electricity at that location. Over time this would provide a signal to investors to indicate whether it would be worth taking load at the location, e.g., building a factory, or supplying load at that location, e.g., building a generator. Prices would be made available every five minutes, allowing the load to make short term decisions as to whether or not they wanted to pay the spot market cost of the electricity, or choose to voluntarily reduce demand.

As Schweppe et al. worded it: "Spot pricing theory provides rules for both optimal short-run decisions and optimal long-run action (investments)". As stated in their paper, the spot market prices would be set by the incremental cost of the next unit, which would send more accurate market place signals in both the short term and the long term. In the short term, i.e., in real time, high prices would drive voluntary load reduction, resulting in less involuntary load shedding as well as less use of expensive oil powered generation. In the long term the prices would provide a true market place value for energy, which would facilitate the entry of new technologies. The goal of deregulation promoted by PURPA would be advanced; generation would be scheduled by optimization, rather then at the discretion of a regulatory authority. The features listed above represent a summary of the attributes of locational marginal pricing (LMP). All that was missing was a practical approach to implement such a system.

2.3 Formulation for a nodal electricity market

In 1988 Schweppe et al. followed up their 1982 paper with a book, *Spot Pricing of Electricity* [63], describing in detail a linear programming model that would allow their electricity spot market to be implemented. This model combined spot market pricing with a simplified AC power flow. A power flow is a model of an electrical system that, when solved, describes how the electrical power flows from the generation to the load. An AC power flow cannot be solved using linear programming, but a DC power flow can. By applying simplifications and assumptions to the AC power flow it can be made to look like a DC power flow. The simplified AC power flow, also known as a DC power flow, can be solved by an LP solver. They showed how a Linear Programming model, which combines bids for load and offers for energy with a DC power flow, can be used to schedule generation to meet load in a way that can be physically dispatched. The load bids and generation offers can only clear if the power flow part of the model can transport the resulting cleared quantities from the generation to the load.

In the same year that Schweppe et al. published *Spot Pricing of Electricity*, Transpower commissioned a report into transmission pricing, with the aim of determining the most appropriate pricing structures for its transmission assets. The report was prepared primarily by Dr Grant Read from the Department of Economics and Operations Research at the University of Canterbury [57]. The report drew on the draft of *Spot Pricing of Electricity* and proposed using a modified version of its methods to solve the power supply problem, with the primary purpose being to extract transmission prices [3]

In 1990 Dr William Hogan from the Harvard Energy Group prepared a paper that proposed using the simplified power flow from *Spot Pricing of Electricity* as a means to making more efficient use of the transmission system [28]. This together with the work of Schweppe et al. was used by Grant Read and Brendan Ring as the basis for the pricing model for the New Zealand electricity market which was developed between 1992 and 1996 [61] [58]. In July 1996 Hogan, Read and Ring presented a paper on the proposed New Zealand LMP electricity market [31]. In October 1996 the New Zealand electricity market became the first to implement LMP. This was a full nodal market including losses and reserves [31].

In April 1998, the PJM[4] electricity market in the USA became the second LMP electricity market [29][34]. The Texas and California markets both underwent electricity market reforms and introduced electricity markets that did not implement LMP; both have since undergone further reform to becoming LMP markets [53][12]. Europe does not have any LMP markets, but LMP has been considered [41]. The Singapore electricity market went live in January 2003, with an LMP market very similar to New Zealand's. China separated generation and transmission in 2002 with the aim of introducing competition into generation. Locational marginal pricing is part of its long term plan [26] [78], but has not yet been implemented due to a slow down in the reforms [50].

In terms of operating the power system, the advantages of the LMP solve are that it produces a *secure* schedule i.e. the LMP result will schedule generation in such a way that transporting the electricity to the load will not overload any of the components that transport the electricity.

In terms of economics, LMP sends transparent pricing signals; the price of electricity at a given location is available to all. If the LMP result consistently produces higher prices in a certain area then this is an indication that this would be a good place to build a generator. LMP sends price signals that reflect the state of supply and demand; when supply is

[3] This was not the first time that operations research had been applied to solving the problems of electricity supply; Grant Read's PhD thesis from 1979 proposed using a linear program to optimise the scheduling of generation in order to maximise the use of hydro-electric resources, i.e., water.

[4] The PJM electricity market covers all or most of Delaware, District of Columbia, Maryland, New Jersey, Ohio, Pennsylvania, Virginia and West Virginia. Parts of Indiana, Illinois, Kentucky, Michigan, North Carolina and Tennessee [16]

plentiful and demand is not then prices are low and vice versa. This encourages demand response.

The disadvantages of LMP are that there is the potential for prices to be volatile and, while prices can always be explained, these explanations are not always straight forward. For example if the LMP solve results in a generation schedule that sends a transmission line to its limit (but, being a secure schedule, not beyond its limit), then this can result in prices at either end of the transmission line diverging sharply. This can also result in electricity prices at some locations varying from the cost of the generation. In extreme cases transmission issues can lead to prices that are negative. These situations will be demonstrated and explained in Section 5.

2.4 SPD: The New Zealand electricity market software

The New Zealand electricity market [5] software that implements the LMP formulation is referred to as SPD. SPD is an acronym for the three distinct time frames covered by the electricity market: Scheduling, Pricing and Dispatch.

The SPD software reads the data necessary to build the LMP model, constructs the LMP formulation, sends this formulation to a simplex solver, then processes the data returned by the solver and presents the results to the electricity market.

SPD determines which generators will generate in order to supply the load. The values used for the load depend on which whether SPD is being run for Scheduling, Pricing or Dispatch. When SPD is being run for *real-time* Dispatch, the load is determined from the actual load at the time that SPD is run, e.g., if SPD is run in Dispatch mode at 6pm at night, the load requirement is based on [6] the total New Zealand load at 6pm. SPD determines which generators will generate to supply this load and the resulting generation quantities are *dispatched* to the generating stations. For pricing purposes the day is divided into 48 half hour trading periods. Early in the morning on the next day the meters are read for all generators and loads. The metered load for each trading period is used as the load input for the Pricing schedule that is run to cover all trading periods in the previous day. The results of the Pricing schedule determine the prices at all of the locations where power enters or leaves the system. These prices, combined with the metered quantities, will determine how much the generators will be paid and how much the purchasers will be charged

Scheduling is the *forecast* of what will happen in *real-time*, i.e., schedules run in "scheduling time" are forecast schedules. Forecast schedules are run every half hour and forecast the generation for the next four hours. A longer forecast schedule is run every two hours, looking forward 72 hours. These forecast schedules help to ensure that when real-time arrives that there will be enough generation available. On the basis of the forecast schedules generators can revise their offers in order to achieve the generation level that they want. Immediately prior to real time purchasers can check the price and if it is too high they may elect to reduce load; large industrial users can achieve this by shutting down plant, retailers by remotely turning off the electric water heaters of domestic customers.

[5] When the New Zealand electricity market started in 1996 it was called the New Zealand Electricity Market (NZEM). Now it is just called the New Zealand electricity market.

[6] Because the load is constantly moving the value that is actually used by SPD is the load at 6pm plus an offset to predict what the load will be in 5 minutes time.

2.5 Summary

In this section we saw that electricity supply has not always been a regulated industry, but that there were good reasons for it to become regulated as electricity use spread and drove economic growth through the twentieth century. We described the problems that arose once electricity growth slowed, the oil crises hit, and new technologies struggled to enter the regulated systems. The subsequent steps taken around the world towards deregulated electricity supply were then laid out. We paid specific attention to the lead up to the introduction of the first LMP market in New Zealand in 1996 and briefly discussed the spread of LMP, before providing a detailed description of the day to day usage of the SPD software that implements LMP in the New Zealand electricity market.

3 Background on Electrical Power Systems

The LMP formulation models the electricity market. In the electricity market the buying and selling of electricity is driven by economic signals, i.e., the cost of generation balanced against the value of supplying load. This economic activity must comply with the constraints of the *power system* that transports the generation to the load. In this section we will describe the components of the power system and explain how the power system is managed by the *System Operator*. Then we will explain the constraints that are used to model the power system within the LMP formulation. The most important of these is the power flow constraint, which is described in detail from the first principals of simple circuit theory.

3.1 Electrical Theory

In order to explain the power system that is modelled by the LMP formulation, we need to present some background electrical theory.

3.1.1 Electrical power and electrical energy

The useful work done by electricity comes from its electric power, measured in watts (W). For example a 100W lightbulb is brighter than a 25W lightbulb because it uses more power. A typical heater is 1000W, more conveniently referred to as 1kW, where kW stands for kilowatt. The power that is used by a city is measured in millions of watts, i.e., megawatts, abbreviated to MW. The MW is the unit of power used by the electricity market and hence by the LMP formulation.

Electric power P is the result of the flow of electrons. This flow has a pressure, which is its voltage V, measured in volts, and its current I, measured in amps. The power of the flow is the product of the voltage V and the current I:

$$P = VI \tag{1}$$

When you turn on a 1kW heater, power flows through the power cord to the heater, where the heater turns 1kW of electrical power into 1kW of heat. What is metered and what you pay for when you use the heater is electrical energy, which is the product of electrical power and time, measured in Watt-hours (Wh). If you leave the 1kW heater on for one hour then you will be charged for 1kWh of energy. If you leave the heater on for only half an hour, turn it off for 15 minutes, then on again for 15 minutes then you will be charged for 0.75kWh of energy. The unit of measurement for domestic electricity consumption is the kWh, referred to on your power bill as a unit of energy. At the level of the electricity market, energy is measured in MWh.

In the electricity market, the amount of energy metered as flowing from a generator will determine how much they are paid. Hence the units for measuring the cost of generation are \$/MWh. Similarly the value of load is also measured in \$/MWh. However, when the power actually flows, the limit that is applied to the physical components, e.g., the limit on the amount of power that a generator can provide, needs to be a limit on the amount at any point in time, not the average quantity. Hence, while the *prices* are energy based, i.e., expressed in \$/MWh, all of the *quantities* reflect the value of the power at any given point in time, i.e., the instantaneous value, measured in MW. This gives rise to some minor issues which we need not concern ourselves with, however it is worth mentioning, in order to explain why we are using \$/MWh for price and MW for quantity.

3.1.2 Ohm's Law and Losses

The LMP formulation includes losses. It is useful to explain losses now, because this will help to explain why the power system puts so much effort into raising and lowering voltages. This also allows us to introduce Ohm's Law which we will need when we describe how the LMP formulation models power flow.

Material that allows electrons to flow through it is referred to as a conductor, e.g., a wire is a conductor. When electricity flows through a conductor some of its power is lost due to the electrons interacting with the conductor itself. The power that is lost, the *losses* of the conductor, is dissipated as heat. The amount of power that is lost is dependent on the *resistance*,R, a physical property of the conductor. Resistance gives rise to the losses and is also the relationship between the current that flows through the conductor and the voltage across the conductor, as described by Ohm's Law:

$$I = \frac{V}{R} \tag{2}$$

Figure 3 shows power being generated and transmitted through a wire to supply a load. We will refer to the wire as *transmission*, because ultimately this example relates to the transmission of power from generation to load. The transmission and the load are represented by resistances, using the symbol -ΛΛΛ-. If the load to be supplied is 1000 watts (say, our 1kW heater) at a nominated voltage of 250 volts, then the current must be 4 amps. To calculate the transmission losses, we need to combine equations (1) and (2). Given that we know the resistance of the transmission is 2 ohms and we know the current flow through the transmission (because it is the same as the current flowing through the load), the transmission losses are:

$$P_{loss} = VI \tag{3}$$

$$= (IR)I = I^2R \tag{4}$$

$$P_{loss} = 4^2 \times 2 = 32W$$

However, if the power was transmitted at 1000V then, by equation (4), the required current to supply 1000W would be 1 amp. Then the losses on the transmission circuit, as per equation (3) would only be $1^2 \times 2 = 2W$, as compared to the $32W$ of losses we had when transmission was at the lower voltage of 250 volts. This shows that the higher the voltage that can be used for transmission, the lower the transmission losses. Hence, when power is transmitted from generators to loads, the voltage is increased before being transmitted and then lowered before being used.

3.2 The Power System

Electricity is produced by *generators*, for example a hydro-electric power station or a wind turbine, and consumed by *loads*, for example your toaster, or an aluminium smelter. The system that makes it possible for the generators to supply electricity to the loads is the *power*

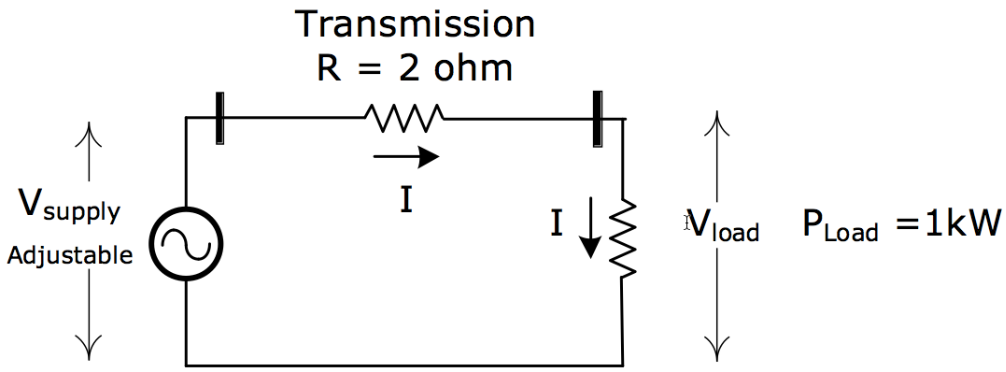


Figure 3: The higher the voltage the lower the transmission losses.

system. The entity responsible for ensuring that the load is *reliably* supplied with generation, i.e., that the power system is *secure*, is the *System Operator*.

Figure 4 shows a diagram of the power system. The LMP formulation includes a model of the portion of the power system shown within the shaded area. The model also includes the constraints necessary to ensure that the LMP result meets the System Operator's requirement for a secure power system. The following sections describe the components of the power system that are included in the LMP model.

3.2.1 Generators, substations, transmission and loads

Generators. The supply of electricity begins at the generator. What the LMP model refers to as a "generator" may be either a single generating unit or a generating station containing more than one generating unit. For example, Huntly Power Station has individual generating units capable of producing 250MW. Each of these is modelled as a generator in the New Zealand electricity market. On the Waikato river, Karapiro Power Station has three generating units with a combined capacity of 90MW. Karapiro Power Station is modelled as a single generator.

Substations. Different generators produce power at different voltages, but generally the voltage is somewhere between 6kV and 40kV [71]. Generators are usually some distance from the places where the power will be used, hence in order to reduce transmission losses (as described in Section 3.1.2), the power from the generator first travels a short distance to a *substation* where its voltage is increased.

At the substation the voltage is increased by a *transformer*. The transformer increases the voltage of the power from the generator to the voltage level of the *transmission network*. The transmission network, or *grid*, is the system of high voltage transmission circuits and substations that transports electricity across the country. In New Zealand the transmission network operates at either 110,000V or 220,000V i.e. 110kV or 220kV [7]. The point at which the

[7] In New Zealand 220kV is the highest voltage on the AC transmission system. AC transmission circuits that have been added in 2013 to bring power into the Auckland area run at 220kV but they have been built so that in the future the circuit could be upgraded to run at 400kV. The DC link the transfers power from between the North Island and South Island runs at a voltage of 350kV.

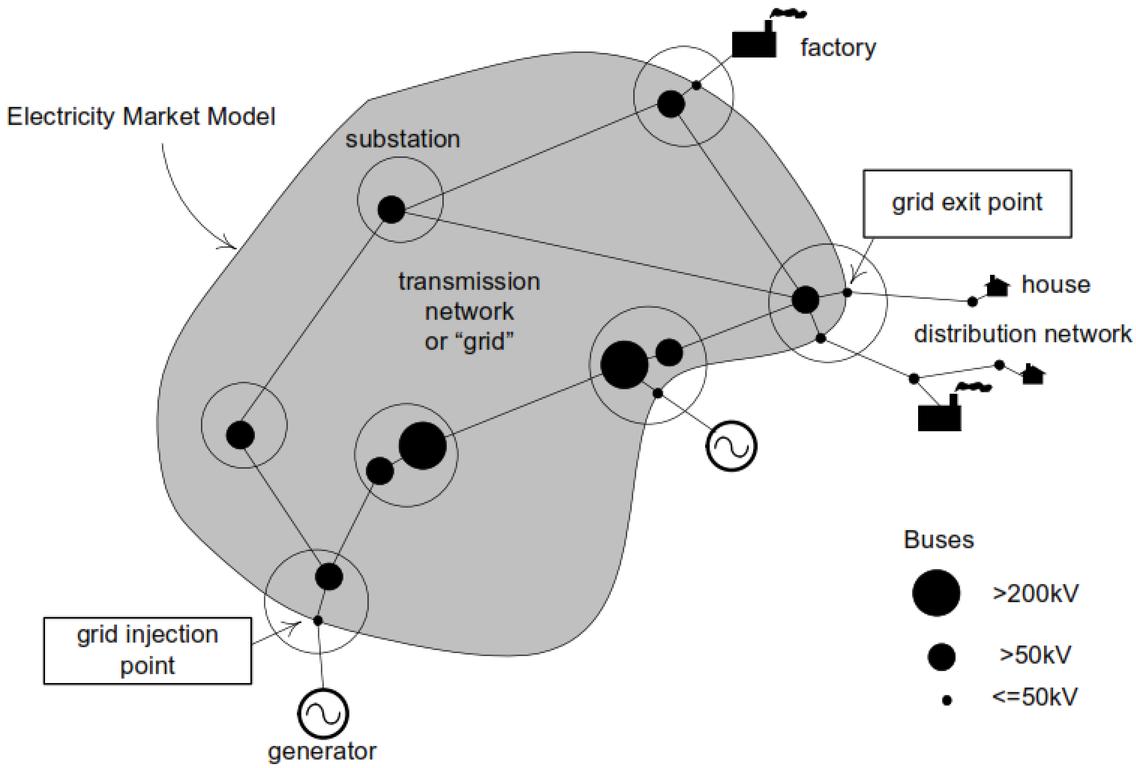


Figure 4: The Power System.

power from the generator leaves the transformer and enters the transmission network is the *grid injection point*. The LMP formulation models the generator as being at the grid injection point. This can be seen in Figure 5 which shows the substation components in detail. The lines connecting the components represent either wires or cables. Each line represents three wires or cables. This is because the power is generated and transmitted as *three phase power*; three wires are necessary to transport three phase power. In each of the three phases the power is alternating current which is changing at different times. The three phase system is used because it works better with generators and motors than single phase, and also means that there is no need for a separate phase to return the current to the generator.

As shown in Figure 5 the components within a substation are connected together by a bus. A bus will be three wires or three metal bars running through the substation, which individual components such as transformers or transmission circuits connect to. Figure 6 shows the "Google Street View" view of one of the 220kV buses at Otahuhu substation in Auckland. Figure 7 is an aerial view of the same corner of the *switchyard* [8], which shows the bus from Figure 6 and a parallel bus[9].

[8]The switchyard is the outdoor area at the substation. The substation will also have an indoor area that contains controls and monitoring equipment. Some substations are completely indoors, i.e., they do not have a switchyard.

[9]From 1992-1994 the author was an outdoor switching operator at Otahuhu substation. This involved planning and executing switching operations in order to isolate equipment and prepare it for maintenance. The electricity reforms were under way and many smaller substations that had previously had dedicated switching

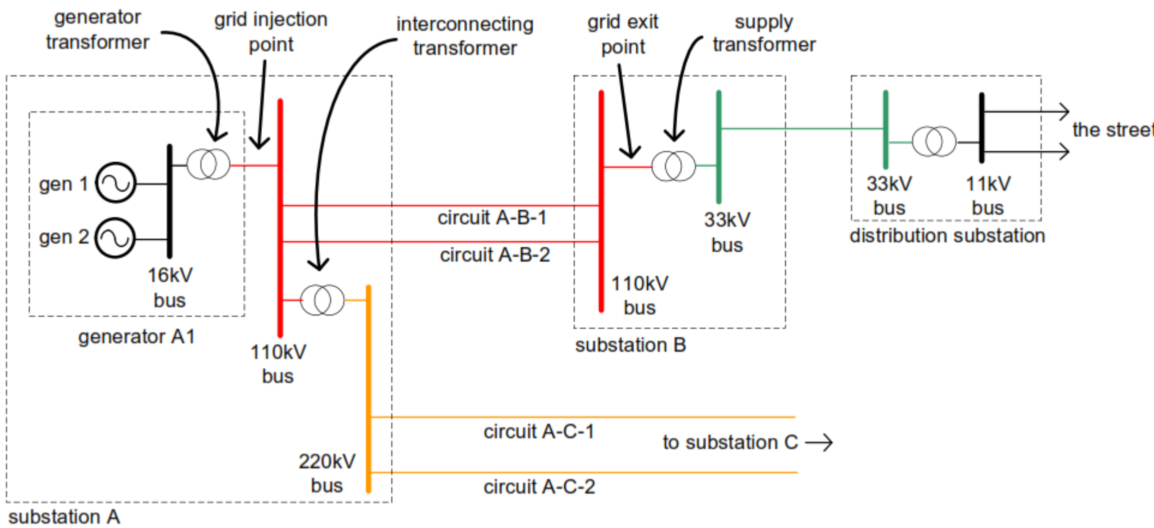


Figure 5: Substation detail.

A substation is also where the transmission voltages may be changed, from the 220kV that travels the long distances to the 110kV that travels not so far. Again the voltages are changed by a transformer, in this case referred to as an *interconnecting* transformer which connects the 220kV bus to the 110kV bus. Figure 8 shows a schematic view of part of Otahuhu substation, taken from the System Operator's schematic view of the electricity market model of the power system [47]. The red lines are 110kV and the orange are 220kV[10]. The overlapping circles represent transformers; T4 and T5 are interconnecting transformers linking the 110kV to the 220kV.

Transmission. The transmission network consists of transmission circuits that carry power between the buses at different substations. In the LMP model a transmission circuit is modelled as a *branch*. In New Zealand the transmission network consists mostly of large pylons strung with wires. Each pylon typically carries two transmission circuits; because they are transmitting three phase power, each transmission circuit consists of three *conductors*, i.e., three wires [11].

The bottom-centre of Figure 7 shows a transmission circuit arriving via a pylon at Otahuhu substation. The circuit connects to a *centre point* between the 220kV buses. There are large switches which allows the power from the transmission circuit to be connected to either bus. The LMP model does not model the switches. If the switch that connects the branch to the

operators were now looked after by roving operators, however at that time Otahuhu still had dedicated switching operators because it is a large substation by New Zealand standards, and important because most of the power for Auckland passes through it.

[10] Because the 220kV buses at Otahuhu are normally connected together, the LMP formulation has only a single 220kV bus in its model of the buses at Otahuhu substation

[11] There also a few places in the New Zealand transmission network where the power is transported by underground cables. In Singapore, where the author worked for the electricity market for four years, all of the transmission system consists of underground cables and all of the substations are in nondescript buildings; it would still be possible to describe the transmission network, but the reader would have little if any personal experience of having observed any of it.

Figure 6: Otahuhu substation with the three phases of the 220kV bus indicated (this picture is from Google Street View).

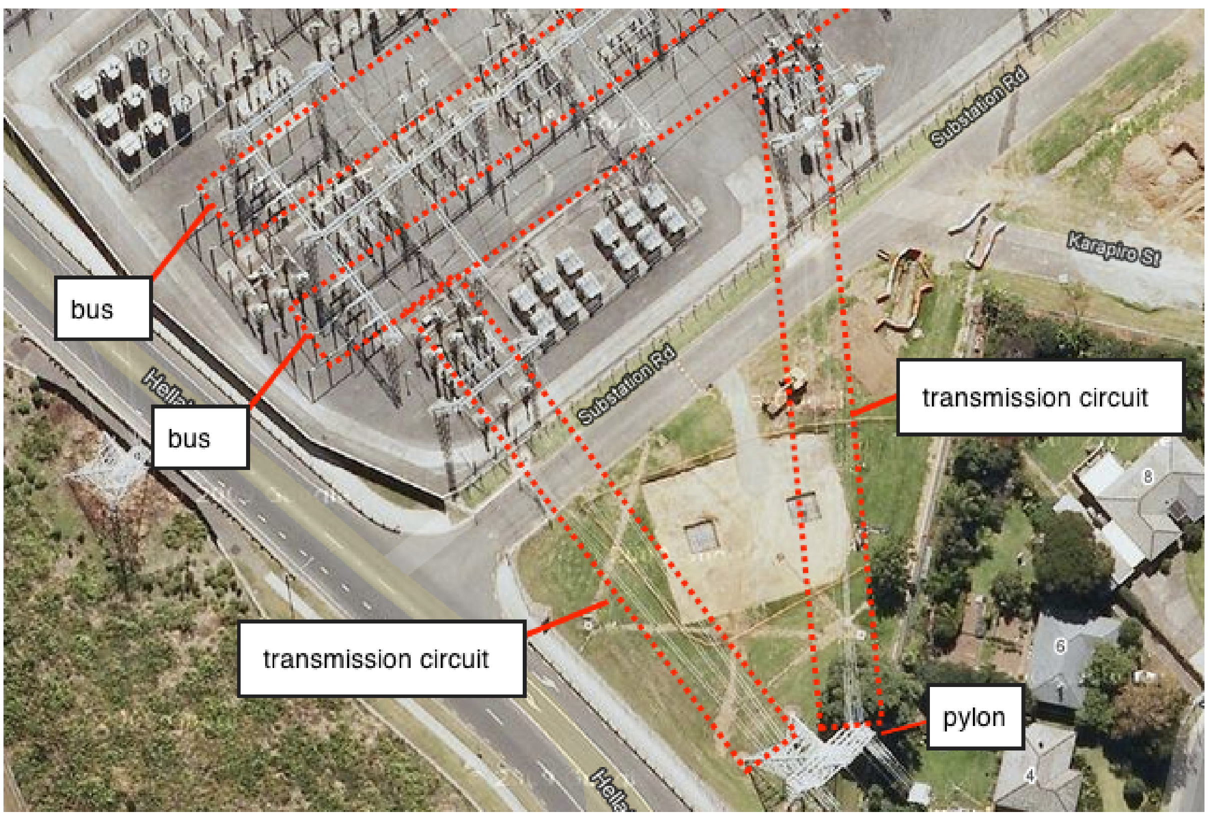


Figure 7: Aerial view of the south-west corner of Otahuhu substation showing transmission circuits (branches in the LMP model) and buses (background image from Google Maps).

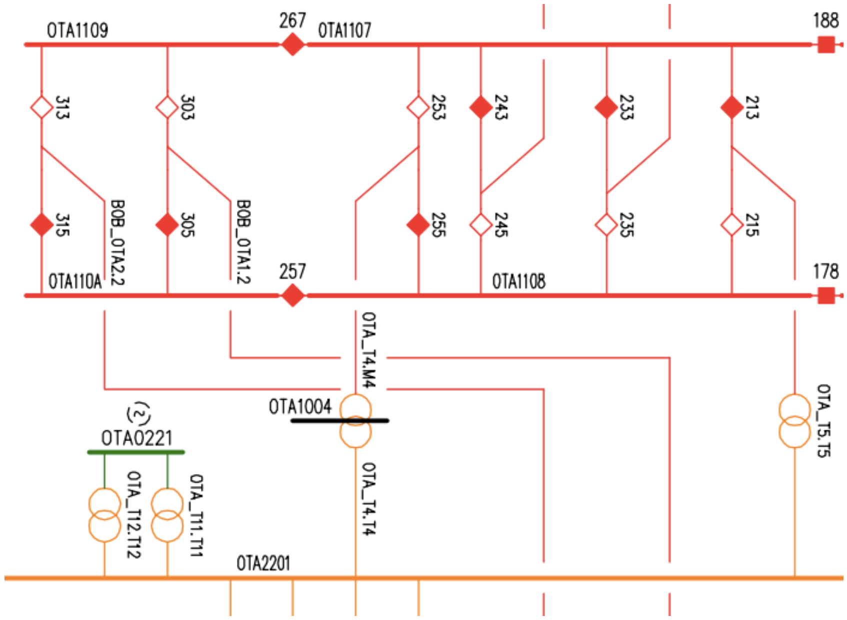


Figure 8: Portion of Otahuhu substation from New Zealand electricity market model [47]
.

bus is opened then in the model we would remove the branch from the model.

Load. A very large load such as an aluminium smelter may take its power at the same voltage as the transmission network, i.e., as a *directly connected load*, but this is the rare exception and for most loads the voltage is lowered from transmission levels before it leaves the substation. As shown in Figure 5, the high voltage is lowered by a *supply transformer* that connects to the bus at the substation. The power enters the transformer's high voltage *primary* side and exits from the transformer's *secondary* side at a lower voltage. The power from the secondary is also connected to a bus. In New Zealand the voltage at this *supply bus* is commonly 33kV. The power from the supply bus leaves the substation and enters the distribution network[12].

In terms of the LMP model, the load is modelled at the point where it leaves the high voltage bus as the substation, i.e., either at the primary side of the supply transformer or at the circuit that provides power to a directly connected load. This is the *grid exit point*.

Figure 9 shows an aerial view of a supply transformer at Otahuhu substation, with a small building indicated for size comparison. The primary of the transformer is connected to the 220kV bus; this connection is the grid exit point. The voltage at the secondary of the transformer is 22kV (33kV is a more common voltage). From the secondary of T12 the

[12]The distribution network distributes the power to distribution substations around a city, or a rural district. In New Zealand the distribution substation lowers the voltage, typically to 11kV. From the distribution substation the 11kV is transported along the streets on power poles, or via underground cables. The 11kV is still three phase power. When this voltage is lowered again (by transformers that are either on power poles, or in small buildings, or the basement of large buildings), a fourth, *neutral*, wire is added. The voltage between phases is now 400V, the voltage between each single phase and the neutral is 240V. Typically it is a single phase and a neutral that enter New Zealand homes, to provide supply at 240V. A large power user, e.g., a large commercial building or a factory will be supplied by three phases and the neutral.

Figure 9: Otahuhu substation showing transformer T12, with a small building indicated for size comparison (this picture is from Google Maps).

power exits the switchyard via underground cables. This can be seen schematically in Figure 8, where the supply bus OTA0221 is coloured green to represent the supply voltage, in this case 22kV (a 33kV voltage would also have been represented by a green line).

3.3 The System Operator

Figure 10 shows an example of the System Operator's control room for the North Island of New Zealand, circa 1995 [13]. The System Operator's responsibility is to maintain *system security* by ensuring that enough electricity is generated to meet the requirements of the load and that the transmission system is capable of transporting the electricity from the grid injection points[14] to the grid exits points[15]. The components of the power system must be run within their limits and bus voltages must be kept within limits of their nominated levels. A secure power system must also be resilient. If a system consists of n items, then if it is able to lose one of these items and still function then it is said to have $n-1$ security. The System Operator must run the power system with $n-1$ security, i.e., such that it can tolerate the

[13] At that time the mapboard of the North Island, shown in the background, was made of mosaic tiles. The red lines in the top half represent the 110kV, below them the orange lines represent 220kV. Haywards substation is at the extreme right hand end of the mapboard, Kaitaia is at the extreme left. The dark rectangles are digital readouts of key values, e.g., the total generation for the North Island. The mapboard has been gone for many years now, replaced by a bank of LCD monitors mounted directly in front of each desk. The space that the mapboard took up has been used for offices.

[14] Generating the power and transporting it to the grid injection point is the responsibility of the generator.

[15] Once the power leaves the grid exit point it is the responsibility of the lines company and the electricity retailers.

Figure 10: The System Operator control room for the North Island Grid, circa 1995 (the author is the one standing). From information booklet Trans Power Control Centres produced in 1995 (when Transpower was written as two words).

loss of any one of its generators or transmission circuits.

3.4 Power Flow

One of the tools for assessing system security is power flow software. Power flow software models the power system; for a given set of load requirements at grid exit points across the power system and a corresponding set of generation quantities at grid injection points, power flow software will model the physics of the power system to determine how the power will flow from the grid injection points through the transmission network to the grid exit points.

The results of a power flow study allow the System Operator to confirm that for a given load scenario and associated generation pattern, the generation is sufficient to meet the requirements of the load and the losses, and that the power system is able to transport the generation to the load without overloading any of the transmission circuits. A simplified version of this power flow modelling is included in the LMP formulation. This ensures that the results of the solution produced by the LMP formulation can be dispatched to generators and result in a secure power system. We will now explain the basis of the power flow constraints that allow the LMP formulation to model power flow. There were several texts that were useful references [39][37][55][3], however none were found that provided a complete explanation, hence we aim to present one here.

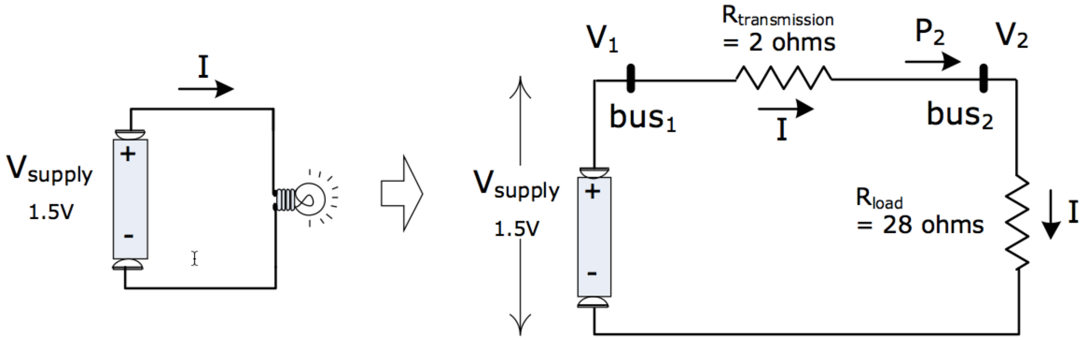


Figure 11: DC power flow example where load is known but load voltage is not.

3.4.1 Direct Current (DC) power flow

The power that is generated in the system that LMP is modeling is Alternating Current (AC). However, the fact that this is AC introduces its own complications. It is easier to introduce the basics of power flow using DC and then cover the AC complications separately.

Figure 11 shows a circuit as you might build yourself with a battery, wires and a bulb. To line up with the power system that we will eventually be modelling we will label each end of the wire as a "bus", and we will label the wire "transmission". In this system we want to calculate the power P_2 flowing into bus 2. By Kirchoff's Current Law [35], the power that flows into the bus must be equal to the power that flows out. The power that flows out is the product of the voltage across the load and the current that flows through the load. In this model we don't know the voltage across the load, we only know the voltage of the battery. We can calculate the voltage across the load, but first we need to know the current. We know the battery voltage, i.e., the *supply voltage*, which is the voltage across the transmission and the load and we know the resistance of the transmission and the load, so we can calculate the current using Ohm's Law:

$$I = \frac{V}{(R_{transmission} + R_{load})} = \frac{1.5}{2+28} = 0.05A \tag{5}$$

Now we use Ohm's Law again, to calculate the voltage across the load:

$$V_2 = IR_{load} = 0.05 \times 28 = 1.4V \tag{6}$$

Hence the power flowing into bus 2 is:

$$P_2 = V_2 I = 1.4 \times 0.05 = 0.07W \tag{7}$$

In the power system, the System Operator does not know the resistance of the load. However, the voltages are managed so that they are kept within limits of a nominated level, the *nominal voltage* of the bus. One of the ways of managing the voltage is to adjust the supply voltage. This is the system illustrated in Figure 12 which differs from Figure 11 in that it is viewed in the same way that the System Operator views the power system; we don't know the characteristics of the load, but we do know the voltage at the *load bus*, i.e. bus 2, and we can adjust the source voltage so that the load bus is at its nominal voltage.

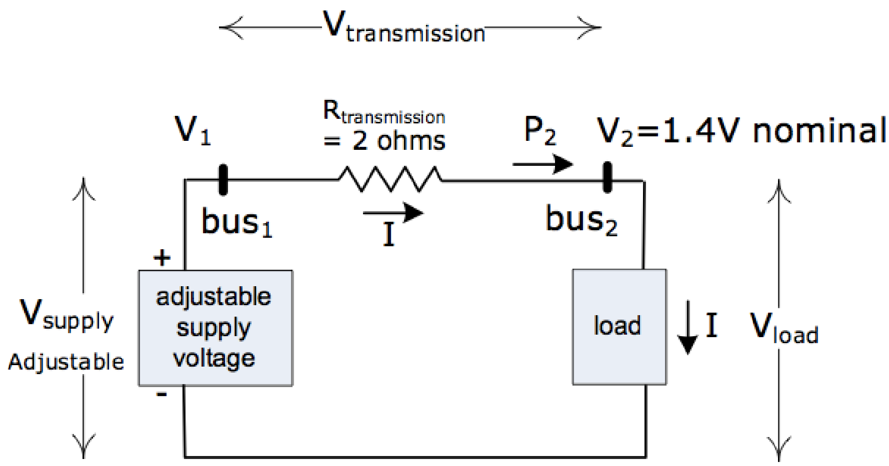


Figure 12: DC power flow example where load is unknown but load voltage is known.

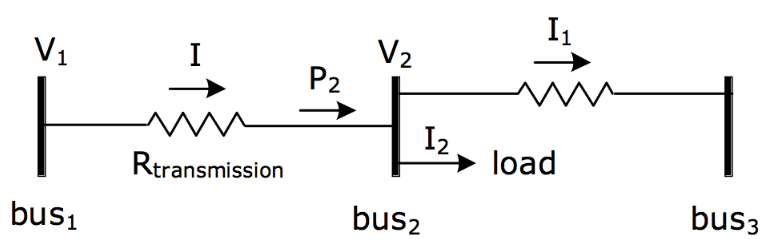


Figure 13: DC power flow where "load" at bus 2 also includes transmission.

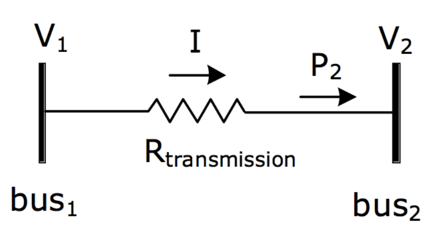


Figure 14: DC power flow general case.

We will say that the nominal supply voltage, i.e. the voltage at bus 2, is 1.4V. We picked this voltage because it lines up with the previous example; hence, we know that we need to adjust the source voltage to 1.5V to achieve 1.4V at the load bus. Everything is exactly the same as the previous example except for the available information; we "don't know" the resistance of the load but we do know the voltage at bus 1 and bus 2 and we do know the resistance of the transmission. Now we proceed to calculate P_2 as follows. First we calculate the current. We don't know the resistance of the load but because *the current flowing through the load is the same as the current flowing through the transmission* we can calculate the current flowing through the load by calculating the current flowing through the transmission:

$$I = \frac{(V_1 - V_2)}{R_{transmission}} \tag{8}$$

Then we can calculate the power:

$$\begin{aligned} P_2 &= V_2 I & (9) \\ &= \frac{V_2(V_1 - V_2)}{R_{transmission}} & (10) \\ &= \frac{1.4(1.5 - 1.4)}{2} = 0.07W & (11) \end{aligned}$$

Although we have explained the DC power flow calculation in terms of a load at bus 2, we can see from equation (10) that it can also be applied to the situation shown in Figure 13, where the power flowing out of bus 2 is due to load and transmission. This leads us to the general case shown in Figure 14 where for any transmission between two buses where we know the voltage at the buses and the resistance of the transmission we can calculate the power flow from the transmission into bus 2 by using equation (10). The only difference between this and the equation for AC power flow is that we also need to include the effect of inductance on AC power.

3.4.2 Inductance

As discussed in Section 3.1.2, a wire has a property called resistance which causes some of the power being transported to be lost as heat. A wire also has a property called inductance. When a changing current flows through a wire, the inductance causes a magnetic field to be set up [23]. The magnetic field produces a voltage which opposes the changing current flow. This opposing voltage is equal to the product of the rate of change of the current and the inductance (L) of the wire [16].

$$V_L = L\frac{dI}{dt} \tag{12}$$

The impact of inductance on DC power flow. Although we want to consider the effect of inductance on our AC power system, it is useful to describe how inductance affects a DC system because this gives some idea of what the physical impact of the inductance is. This will be less obvious when we get to the AC case. Also the concepts that we introduce here will be used later.

[16] Inductance is measured in Henries (H).

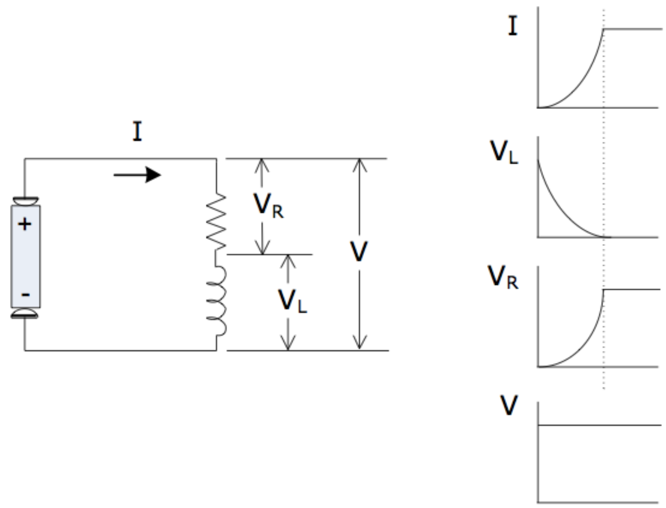


Figure 15: Impact of inductance on a DC circuit.

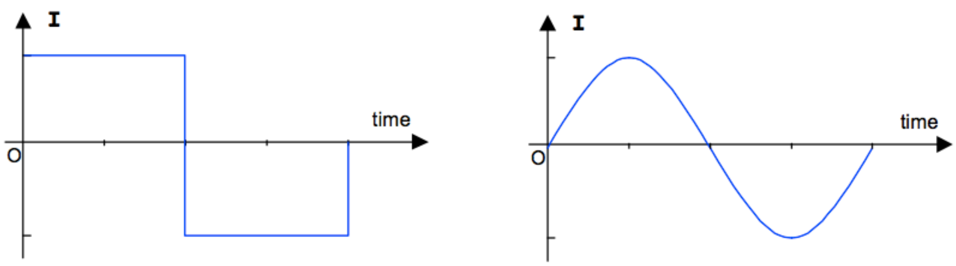


Figure 16: (a)Square Wave (b)Sine Wave.

The impact of inductance on a DC circuit is shown in Figure 15. The resistance and the reactance (symbolised by  of the wire are modelled as series components. When the battery is connected, the rate of change of current is greatest; it was zero and now suddenly it is not. Therefore at this point the opposing voltage across the inductor is also at its greatest, i.e., equal to the source voltage. The opposing voltage slows the rate of change of the current, this in turn lowers the opposing voltage; hence the current reaches its final value which is determined by Ohms Law as $I = \frac{V}{R}$. In steady state the voltage across the inductor is zero. It is important to note that at all times the sum of the voltages across the components in the circuit is equal to the source voltage; this is Kirchoff's Voltage Law [36].

3.4.3 Alternating Current (AC)

If the battery shown in Figure 11 were turned around then the light would continue to shine. The current would be flowing in the other direction, but because the voltage was also reversed the power flow would still be positive. If we could reverse the battery in zero time then we would obtain the square wave shown in Figure 16(a). This is an example of an Alternating Current (AC).

The AC that is produced by AC generators and flows through the power system to boil your kettle is also changing direction, but it is a sine wave, as shown in Figure 16(b). If we

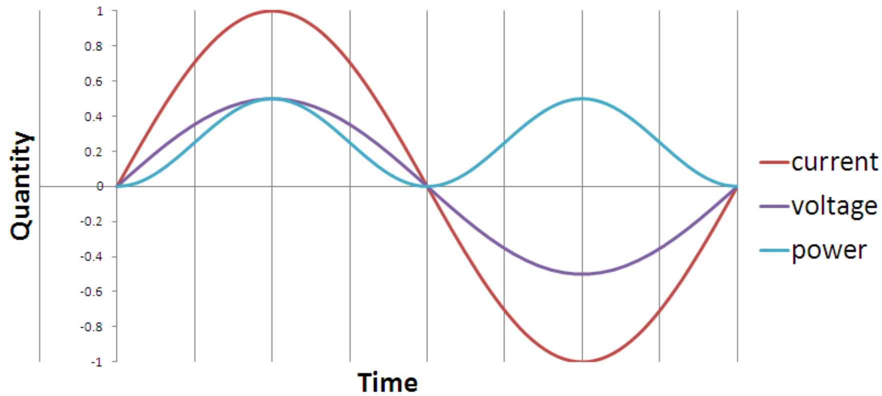


Figure 17: AC current, voltage and power for a circuit with only resistance

replaced the battery in Figure 11 with an AC generator and assumed that the wire has no inductance, then the current and the voltage would reverse at the same time and the product of these, the power waveform, would appear as shown in Figure 17. The power delivered is the area under the power waveform, which is always positive, hence the power is always flowing from the generator to light the bulb. However, this serves to illustrate a hypothetical extreme, as every wire does actually have some inductance.

The impact of inductance on AC power flow. When an AC current flows through an inductor, Equation (12) has more of an impact than it did on the DC circuit. Because AC current is always changing, there is always an induced voltage present across the inductance. As per Equation (12) this voltage is proportional to the rate of change of the current. The current is a sine wave, differentiation (rate of change) of a sine function gives a cosine, hence the voltage across the inductance is a cosine wave, or in other words, a sine wave that starts 90°before the sine wave of the current flow.

Figure 18 shows an AC circuit with a resistance and reactance, where ⏦ represents a source that produces AC power, e.g., an AC generator. Figure 19 shows the waveforms of the voltage and current for this circuit. The current that flows through the resistance and the reactance is the same. The waveform of the voltage across the resistor follows the current, i.e., the current flowing through the resistor and the voltage across the resistor are in phase. The waveform of the voltage across the inductor leads the current through the inductor by 90°. The total voltage is the sum of the V_R and V_L waveforms.

Figure 20 shows the power calculations for an AC circuits that only has inductance. The power flows into the inductor for half the cycle and then back to the generator for the other half. The sum total of the power transferred to the inductor is zero. Figure 21 shows the power waveform for the circuit shown in Figure 18. The total power flow shows that the impact of the inductance is that although over-all the power flow is positive, some of the power flow is flowing from the generator to the load and then back again.

Phasor representation of AC waveforms. Looking at Figure 19 we can see that the waveforms for V_R and V_L have the same magnitude and by observation the waveform of the total voltage V leads the current by 45°. To work out exactly what this waveform is we use pha-

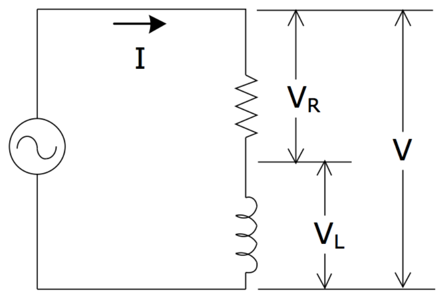


Figure 18: AC circuit with resistance and reactance

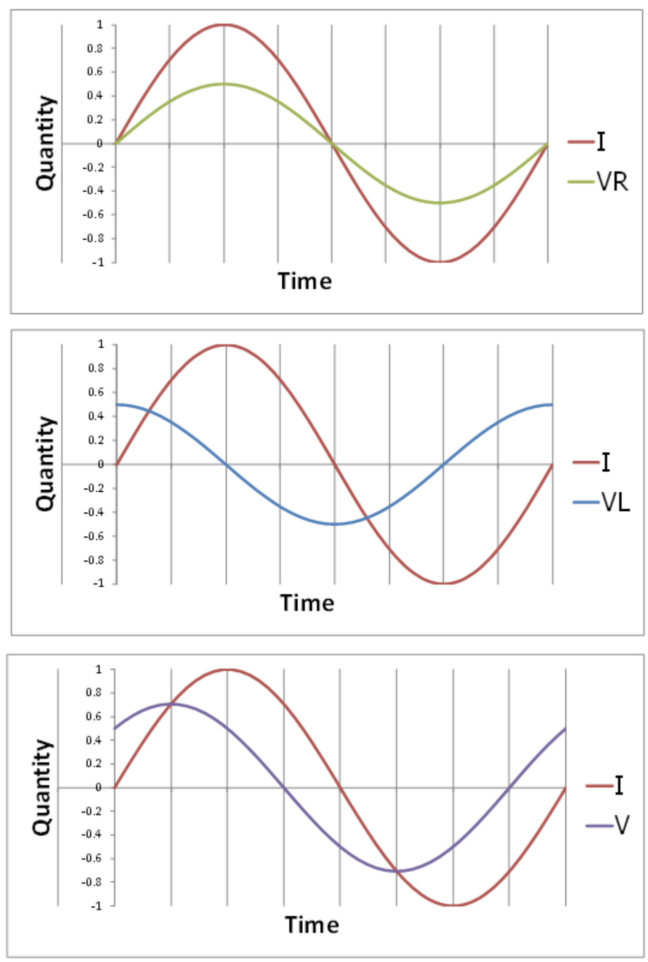


Figure 19: Voltage and current waveforms for an AC circuit with resistance and reactance

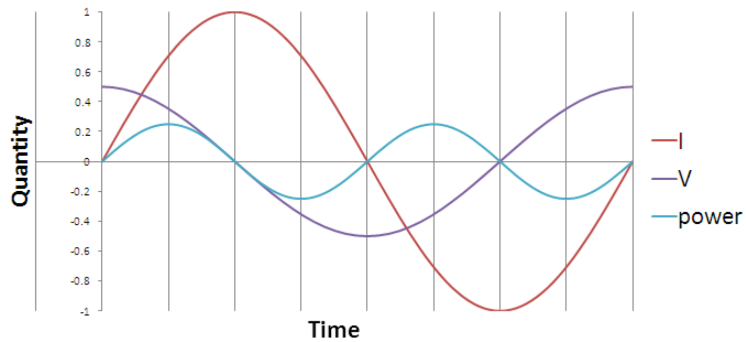


Figure 20: Power waveform for an AC circuit with only reactance

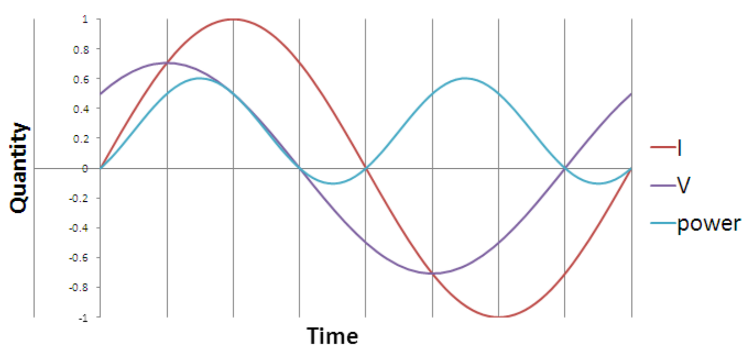


Figure 21: Power waveform for an AC circuit with equal resistance and reactance

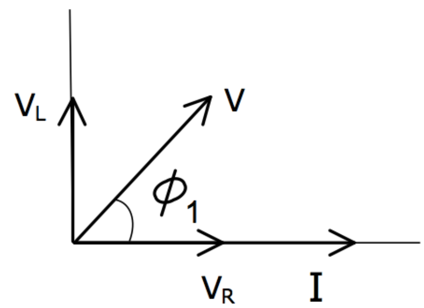


Figure 22: Phasor representation of voltage and current

sors [75] to represent the waveforms. Taking the current with an angle of 0°as the reference and with the V_R in phase with the current and V_L leading the current by 90°we can draw Figure 22. With V_R and V_L having the same magnitude, V leads I by 45°.

Reactance. To calculate actual values we need to know how to calculate the voltage and current in the inductor. From Ohm's Law we know the magnitude of the voltage across the resistance is $V = IR$. Equation (12) describes how the voltage across the inductor is the product of the rate of change of the current and the inductance, but it does not give us an actual figure. For this we need to know the reactance of the inductor. The reactance, X is the quantity that provides the relationship between the inductor's voltage and the current in an AC circuit:

$$X = 2\pi f L \tag{13}$$

where f is the frequency of the AC waveform, i.e., how many times per second the sine wave repeats its cycle. Reactance relates the voltage and current in an inductor in the same way that resistance relates the voltage and current in a resistor. Reactance is measured in ohms, the same units as resistance.

Modelling resistance and reactance. Multiplication by an imaginary number, i.e. a number expressed in terms of units of j the imaginary unit[17], has the effect of rotating a phasor 90°counter-clockwise [74], i.e., advancing the phase angle by 90°. We can use this to calculate the voltage phasor across an inductor from the current phasor and the reactance:

$$\vec{V_L} = jX\vec{I} \tag{14}$$

where $vecV_L$ is the phasor representing the voltage across the inductace and $\vec{I}$ is the phasor representing the current. Regardless of the angle of the current phasor, Equation (14) ensures that the resulting voltage across the inductance leads the current by 90°. This leads us to model the resistance and the reactance of the circuit as $R + jX$, as the imaginary unit assigned to the reactance will always result in the voltage across the inductance leading the current by 90°.

[17]The imaginary unit is also expressed as i, in electrical engineering j is used in order to avoid confusion with the representation of current using i.

Equation (14 is expressed in terms of phasors. To calculate results without having to draw phasor diagrams we express the voltage and current phasors in terms of their *real* and *imaginary* components. The real component is the 0°reference, while the imaginary component is 90°ahead of the reference. The imaginary component is indicated, and expressed mathematically, by assigning it the imaginary unit j. We calculate the real and imaginary components from the magnitude of the phasors as follows.

$$\vec{V} = Vcos\phi_1 + jVsin\phi_1 \tag{15}$$

$$\vec{I} = Icos\phi_2 + jIsin\phi_2 \tag{16}$$

The angle of the voltage, ϕ_1, and the angle of the current, ϕ_2, are expressed relative to a current that has an angle of 0°. The magnitude and angle of the voltage is set by the generator [68], therefore the current will be determined by the source voltage and the total resistance and reactance of the circuit:

$$\vec{I} = \frac{Vcos\phi_1 + jVsin\phi_1}{R + jX} \tag{17}$$

The complication of having $R + jX$ in the denominator is resolved as follows:

$$\frac{1}{R+jX} = \frac{1}{R+jX} \times \frac{R-jX}{R-jX} = \frac{R}{R^2+X^2} + \frac{-jX}{R^2+X^2} = G + jB \tag{18}$$

where G is the *admittance* and B is the *susceptance*:

$$G = \frac{R}{R^2+X^2} \tag{19}$$

$$B = \frac{-X}{R^2+X^2} \tag{20}$$

Hence, the equation to calculate the current is:

$$\vec{I} = (Vcos\phi_1 + jVsin\phi_1)(G + jB) \tag{21}$$

AC power. As with the voltage and current, AC power also consists of a real and imaginary component. The real component represents the product of the current together with the voltage projected onto the current. The imaginary component of the power represents the product of the current together with the component of the voltage that is 90°out of phase with the current.

As we saw in Figure 17, the product of the in-phase current and voltage waveforms will produce power that is always positive, i.e., power that is consumed by the load. This is the *real* or *resistive* power. The product of the 90°out of phase waveforms will be power that flows from the generator and then back, for a net transfer of zero. This is the *imaginary* or *reactive* power. The symbol for AC power is S, the real power is P and the reactive power is Q:

$$\vec{S} = \vec{P} + j\vec{Q} \tag{22}$$

For the electricity market, and hence the LMP model, we are only modelling real power flow, i.e., the P component of Equation (22).

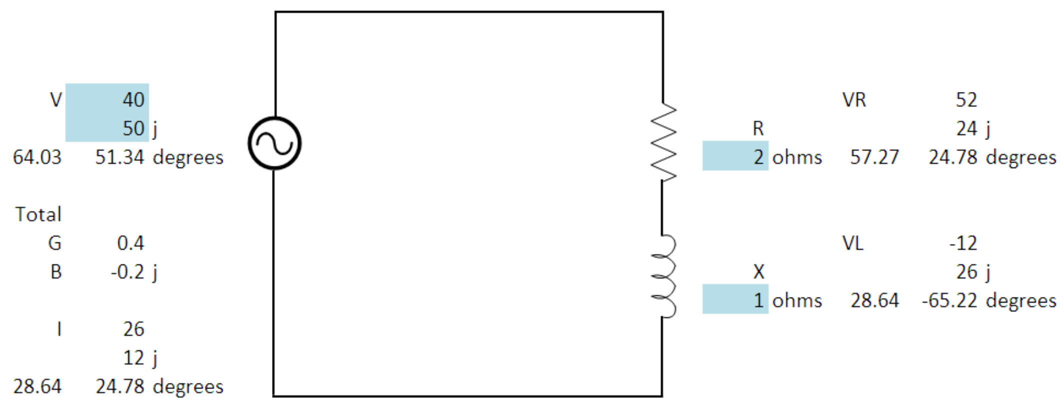


Figure 23: Example of AC circuit with current calculated from source voltage and circuit R and X.

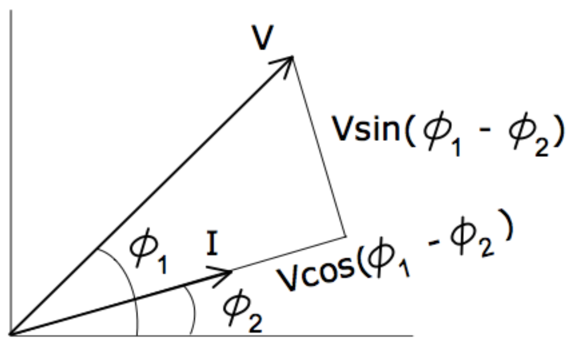


Figure 24: Voltage and current phasors, showing the components of the voltage that we want to multiply by the current.

AC power calculation. With the above definitions in place we can now look at calculating AC power flow. Figure 23 shows an example of a circuit where we have used Excel to calculate the current from the source voltage and the total R and X, using Equation (21). This serves to demonstrate how we arrive at the situation where the voltage and current that we want to multiply together both have a non-zero angle.

Figure 24 shows the components of the voltage that we want to multiply by the current. We may think that we can multiply the components of the phasors together as we do when we multiplied the voltage phasor by the $G + jB$ phasor to get the current. However, in that case the real component G has a 0°angle. In our example I does not have a 0°angle. If we multiply the voltage phasor by the current phasor we get:

$$\begin{aligned}\vec{V}\vec{I} &= (Vcos\phi_1 + jVsin\phi_1)(Icos\phi_2 + jIsin\phi_2)\\ &= VI((cos\phi_1 cos\phi_2 - sin\phi_1 sin\phi_2) + j(sin\phi_1 cos\phi_2 + cos\phi_1 sin\phi_2))\end{aligned} \tag{23}$$

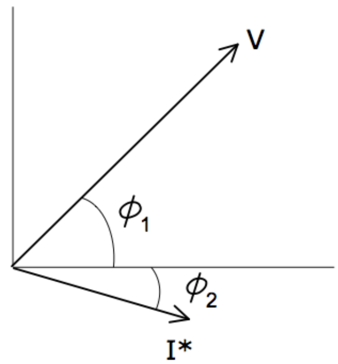


Figure 25: Voltage phasor and complex conjugate of the current phasor.

By comparing this to the following trigonometric formulas [67]:

$$sin(A+B) = sinAcosB + cosAsinB \tag{24}$$
$$cos(A+B) = cosAcosB - sinAsinB \tag{25}$$
$$sin(A-B) = sinAcosB - cosAsinB \tag{26}$$
$$cos(A-B) = cosAcosB + sinAsinB \tag{27}$$

We see that what we have arrived is the following result:

$$\vec{V}\vec{I} = VI(sin(\phi_1+\phi_2) + jcos(\phi_1+\phi_2)) \tag{28}$$

Comparing this to Figure 24, we can see that this is not what we want; we do not want a formula that adds the angle of the voltage and the current, we want a formula that projects the voltage onto the current, and this must be based on the difference between the two angles, not the sum. What it turns out we need to do is reverse the sign of the imaginary component of the current, i.e., multiply the $jIsin\phi$ component by -1. This will give us the phasor diagram shown in Figure 25. The phasor that is obtained by reversing the sign of the imaginary component is referred to as the *complex conjugate* and indicated by applying an asterix to the original phasor. Now when we do the maths:

$$\vec{V}\vec{I^*} = (Vcos\phi_1 + jVsin\phi_1)(Icos\phi_2 - jIsin\phi_2) \tag{29}$$
$$= VI((cos\phi_1cos\phi_2 + sin\phi_1sin\phi_2) + j(sin\phi_1cos\phi_2 - cos\phi_1sin\phi_2)) \tag{30}$$

Comparing this with formulas (24) to (27), we find that we have the result we require, i.e., the AC power is calculated based on the phase angle difference between the voltage and the current:

$$S = \vec{V}\vec{I^*} = VI(cos(\phi_1-\phi_2) + jsin(\phi_1-\phi_2)) \tag{31}$$

We are now ready to calculate the power flowing into bus 2 in Figure 26. Even though we are only interested in the real power component, we need to calculate the full AC powerflow result before we can separate this out. We proceed by combining the AC equivalent of the

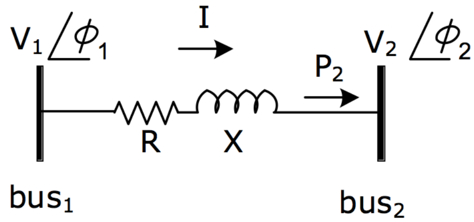


Figure 26: AC power flow example.

DC powerflow shown in Equation (10), with the current calculated by combining Equations (17),(19) and (20):

$$\vec{S} = \vec{V}\vec{I}^{*} = \vec{V}_1[(\vec{V}_1 - \vec{V}_2)(G + jB)]^{*} \tag{32}$$

Because the powerflow calculation uses the complex conjugate of the current, we have also applied the complex conjugate to the calculation that replaces the current, i.e., $[(\vec{V}_1 - \vec{V}_2)(G + jB)]^{*}$. To expand this out we will be using the following definitions and rules for complex conjugates [69]:

$$\vec{z} = x + jy \tag{33}$$

$$\vec{z^{*}} = x - jy \tag{34}$$

$$\vec{z}\vec{z^{*}} = z^2 \tag{35}$$

$$(\vec{z}_1 - \vec{z}_2)^{*} = \vec{z}_1^{\,*} - \vec{z}_2^{\,*} \tag{36}$$

$$(\vec{z}_1\vec{z}_2)^{*} = \vec{z}_1^{*}\vec{z}_2^{*} \tag{37}$$

Applying these to Equation (32) and also considering Equations (15), (26) and (27):

$$\vec{S}_2 = \vec{V}_1(\vec{V}_1 - \vec{V}_2)^{*}(G + jB)^{*} = \vec{V}_1(\vec{V}_1^{\,*} - \vec{V}_2^{\,*})(G - jB) \tag{38}$$

$$= (V_1^2 - \vec{V}_1\vec{V}_2^{*})(G - jB) \tag{39}$$

$$= (V_1^2 - (V_1 cos\phi_1 + jV_1 sin\phi_1)(V_2 cos\phi_2 + jV_2 sin\phi_2)^{*})(G - jB) \tag{40}$$

$$= (V_1^2 - (V_1 cos\phi_1 + jV_1 sin\phi_1)(V_2 cos\phi_2 - jV_2 sin\phi_2))(G - jB) \tag{41}$$

$$= (V_1^2 - V_1V_2(cos(\phi_1 - \phi_2) + jsin(\phi_1 - \phi_2))(G - jB) \tag{42}$$

By multiplying through and separating out the real components we arrive at an equation for the real power flow component:

$$P_2 = GV_1^2 - GV_1V_2cos(\phi_1 - \phi_2) + BV_1V_2sin(\phi_1 - \phi_2) \tag{43}$$

From Equation (43) the simplified AC powerflow is obtained by making several assumptions. The first assumption is that, for a large transmission circuit, the reactance of the line is significantly larger than the resistance, hence G can be ignored giving:

$$P_2 = BV_1V_2sin(\phi_1 - \phi_2) \tag{44}$$

We also assume that all values are normalised to a per-unit system (as described in Appendix A) where voltages that are at their nominated levels have a value of 1.0, e.g., we

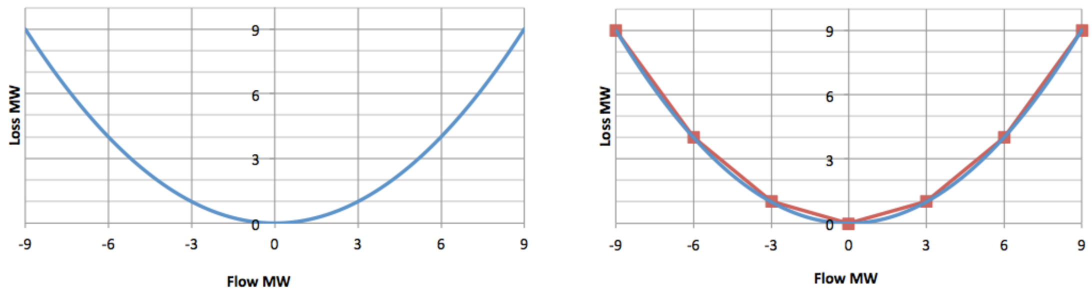


Figure 27: Flow-loss curve and piece-wise linear approximation.

assume that a bus with a nominated voltage of 110kV is actually at 110kV, in which case its per-unit voltage is 1.0. This allows V_1 and V_2 to be replaced with 1.0. The final assumption is that difference in phase angles is small, hence $sin(\phi_1 - \phi_2) \approx (\phi_1 - \phi_2)$. With these assumptions in place, the formula for the powerflow into bus 2 with a phase angle of ϕ_2 from bus 1 with a phase angle of ϕ_1, via a transmission line with a susceptance B is:

$$P_2 = B(\phi_1 - \phi_2) \tag{45}$$

3.5 Branch Losses

Schweppe et al. [63] show how the branch losses can be calculated from Equation (43), and some approximations, to arrive at:

$$P_{loss} = P_2^2 \times R \tag{46}$$

where R is the resistance of the branch and P_{loss} represents the branch losses. As this is a quadratic function, in order to be able to include it in the LP it must be expressed as a set of linear equations. The parabola that results from Equation (46) is shown in Figure 27. To linearise this curve it is first represented as a series of straight line flow-loss *segments*, i.e., a piece-wise linear approximation.

Figure 28(a) shows the segments that are calculated directly from the loss formula. The total flow on each segment is determined by dividing the maximum flow of the branch by the desired number number of segments. The losses for the end points of each segment are then calculated from the flow at that point and the resistance of the branch, as per Equation (46). Only positive flow segments are used by the LMP app because branch flows are modelled as always positive, with each single branch effectively being represented in the model by two branches in opposite directions, each with positive flows .

Figure 28(b) represents the segments as they appear in the LMP model. Each flow-loss segment is modelled as an individual constraint, with a minimum flow of zero and a maximum flow equal to the total flow on the segment, which is the same for each segment. The only difference between the segments is that those that represent the higher flows will have higher losses.

A constraint in the LMP model restricts the total flow on each branch to be the sum of the flows on its flow-loss segments. Hence, the model must schedule flow on the flow-loss

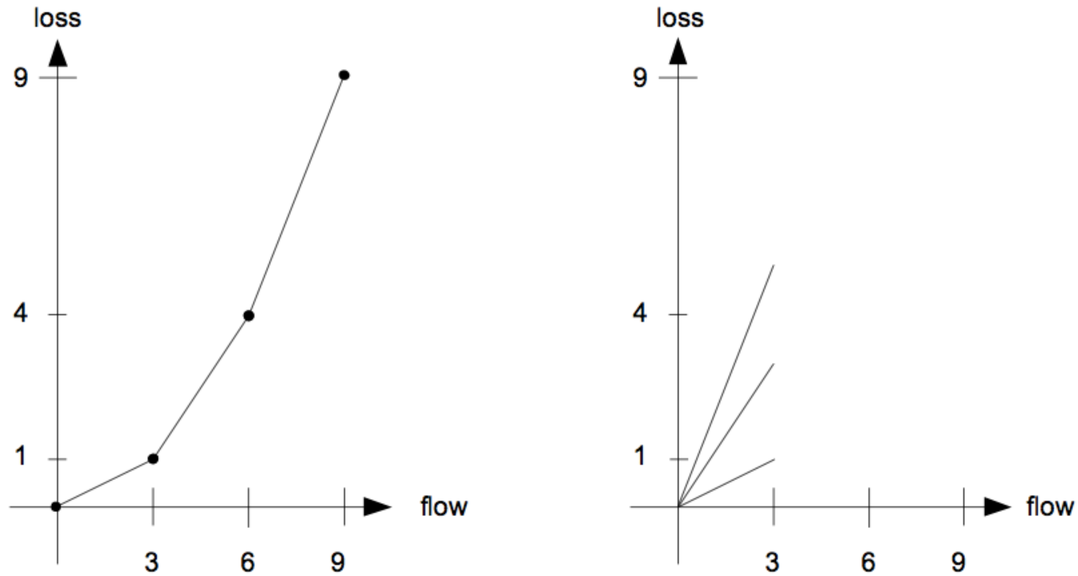


Figure 28: Flow-loss curve: (a) As calculated (b) As modelled.

segments in order to schedule branch flow. Another constraint links the flow on the flow-loss segment to its loss. Hence, branch flow leads to segment flow which leads to losses. Because losses usually have a negative impact on the objective value (see Section 5.4.2 for a situation where this is not the case), the model will clear the flow-loss segments associated with the lower levels of flow first, because they have a lower level of losses. This system of modelling works because the parabola is convex and losses have a negative impact on the objective. More complex curves require a piece-wise linear approximation that uses weights associated with each segment [11].

For *each* branch, the modelling of losses using six loss segments (three in each flow direction) adds 11 new constraints to the formulation. In the New Zealand and the Singapore electricity markets, which are both relatively small compared to those in the USA[18], the branch losses are modeled, in most other LMP electricity markets they are not. There are some differences between the way that New Zealand and Singapore model losses [17][44]:

- New Zealand uses segments with associated flow-loss ratios whereas Singapore uses weights associated with flows and losses, and a constraint that ensures the sum of the weights is 1.0 [11]. The LMP app follows the New Zealand system.
- New Zealand estimates the points on the loss curve by using least squares to minimise the error whereas Singapore simply takes points directly from the curve. The LMP app follows the Singapore system.
- New Zealand assigns all the losses to the receiving end of the branch whereas Singapore assigns half the losses to each end of the branch. The LMP app follows the

[18]The all time peak demand for the New Zealand electricity market is 6,461MW in 2007 [8]. The five LMP electricity markets in the USA have all time peaks as follows: California (CAISO) 50,270MW in 2006; Midwest (MISO) 110,500MW in 2006; New England (ISO-NE) 28,130MW in 2006, New York (NYISO) 33,035 in 2006; PJM 144,644MW in 2006; Texas (ERCOT)65,700MW in 2010. [16]

Singapore system.

The difference in bus prices across the country caused by the modelling of losses in the New Zealand electricity market can be observed on the free to air website of the Wholesale Information Trading System for the New Zealand electricity market [25].

3.6 Summary.

In this section we have described the power system that is modelled by the LMP formulation. The power system enables power to flow from generators to loads. We then laid out in detail how this physical power flow can be modelled using mathematical constraints. Within the LMP formulation the power flow will be modelled by applying these constraints to a model that represents the main physical components of the power system; generators, buses, transmission circuits (branches) and loads. The power flow constraints ensure that the generation result produced by the LMP formulation can be physically implemented on the real power system, with generation sufficient to meet load plus losses and without overloading any of the components.

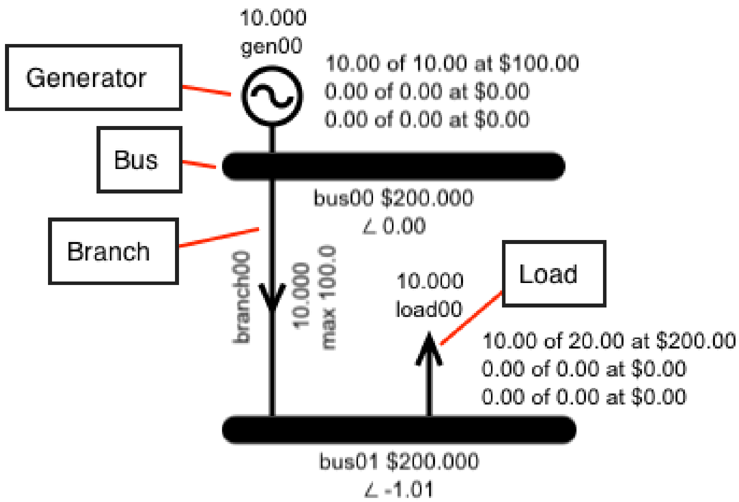


Figure 29: Components of the LMP model.

4 The LMP Model Formulation

This section describes the Linear Programming (LP) model of LMP. First we will describe the physical components that are modelled by the LMP formulation. This is followed by a description of the bids and offers that constitute the economic components of the model. We then explain the variables and parameters relating to the components and present them in the form that they will appear in the LP formulation. We then describe the constraints related to the formulation and present the LP formulation of the LMP model. This is followed by a comparison of the LP formulation of LMP with the LP formulation of the minimum cost network flow problem.

4.1 Components of the LMP model

Figure 29 shows an LMP model created by the LMP app, displaying the components of the power system (described in Section 3.2) that are modelled by the LMP formulation:

- A *bus*, represented by the symbol ▬, is a common connection point, i.e., generators, branches and loads all connect to buses. In the physical system a bus is located within a substation.
- The power is transmitted between buses by *branches*, represented by the symbol →—. The arrow head on the branch symbol indicates the direction of the flow result. In the physical system the branch is either a transmission circuit that connects buses in different substations, or a transformer which connects buses at different voltages within the same substation. Branches have associated losses, i.e., power is lost as it flows through the branch. Because the losses in a branch are quadratic in nature, to include them in the LMP formulation, which will be solved by *linear* programming, the quadratic loss curve must be modelled using a series of flow-loss segments, as described in Section 3.5.

Table 1: Bids and Offers.

Block	Quantity	Price
1	1	\$0.50
2	1	\$4.00
3	1	\$10.00

(a) Bids

Block	Quantity	Price
1	0.5	\$1.00
2	3	\$2.00
3	1	\$15.00

(b) Offers

- A generator, represented by the symbol , produces electrical power, as described in Section 3.2.1. A generator submits offers to the effect that they will generate a quantity of power, provided they receive more than a specified price, as described in Section 4.2.
- A load, represented by the symbol $\uparrow$, consumes electrical power, as described in Section 3.2.1. The load submits bids to the effect that they will purchase power, provided they pay no more than a specified price, as described in Section 4.2.

4.2 Bids, Offers and the LMP Objective Function

In the electricity spot market proposed by Schweppe, et al. [63] (see Section 2.3), purchasers bid to buy electricity, based on the value that they place on the commodity (the electricity). Generators offer to sell electricity based on the long run marginal cost of the generating plant, i.e., a combination of the cost of production and the return on the investment, based on the expected life of the plant and how often the plant is expected to run [66]. The bids and offers are made in terms of price-quantity blocks, as shown by the example in Table 1. A bid will be *cleared* if the price that the purchaser will be charged is less than or equal to the bid price. An offer will be *cleared* if the price that the generator will be paid is greater than or equal to the offer price.

The objective of the LMP model formulation is to maximise the value of the load that is supplied with electricity, while minimizing the cost of the generation that produces the electricity:

$$\text{Maximise} \quad \sum_{bids} Bid_{QuantityCleared} \times Bid_{Price} - \sum_{offers} Offer_{QuantityCleared} \times Offer_{Price} \quad (47)$$

If there were no need to model the transmission of the power between the generation and the load, the example shown in Table 1 would result in the bids and offers clearing as shown in Figure 30 (which was produced by the LMP app). The \$0.50 bid block did not clear because none of the offers have a price less than or equal to \$0.50. The \$2 offer block only cleared up to the point where bids priced above \$2 are exhausted. The \$15 offer block did not clear at all because no bids have a price greater than or equal to \$15. The marginal price at the bus is \$2 because this is the price of the partially cleared generation (how this price comes about will be explained more fully in Section 5).

Because the LMP formulation also models the transmission it is not usually as simple as directly matching bids to offers in this way. The price that the purchaser will pay and the price that the generator will be paid is the *locational* marginal price, i.e., the price at the relevant grid exit or grid injection point. This price will include the impact of the losses that

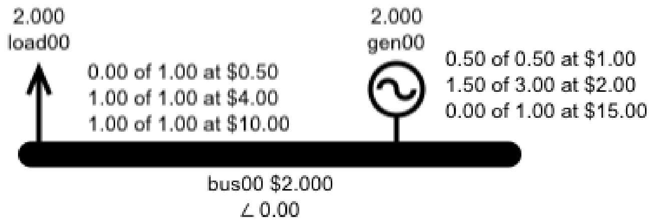


Figure 30: Bids and offers clearing.

occur as a result of transporting the generation to the load, as well as the impact of any other constraints that model the physical reality of the transmission, such as the maximum flow limits on branches.

4.3 Linear Programming Formulation of LMP

To explain the formulation we need to add detail to the components in the form of their parameters, variables and the constraints that the formulation applies.

Parameters. Figure 31 shows the parameters of the components. The parameters are described as follows:

- Both the bids and offers are modeled as a set of blocks associated with either a load (for bids) or generator (for offers). Each block has a price and a quantity, where the quantity is the maximum that can be cleared by the block.
- The power from the cleared generation block(s) flows along the branch(es) to the cleared bid block(s). The flow on a branch cannot exceed the maximum flow rating of the branch.
- In order to determine how the power flows through the system, the formulation needs to include a power flow constraint, as described in Section 3.4. One of the requirements of the power flow constraint is that the system has a reference bus defined. The identity of the reference bus is a parameter of the system being modelled.
- To determine the power flow through a particular branch the power flow constraint requires the branch susceptance, which is a parameter of the branch.
- When the power flows through the branch, some of it is lost due to the resistance of the branch. The loss is modeled by flow-loss segments, as described in Section 3.5. A set of loss segments is associated with each branch. Each flow-loss segment has an associated flow-loss ratio, i.e., how much loss will occur on the segment for a given flow on the segment. Each flow-loss segment also has a maximum flow. There is a constraint associated with the branch flow that will force it to be equal to the sum of the flow-loss segment flows, hence, in order for there to be branch flow there must be segment flow, and therefore losses.

Variables. Figure 32 shows the variables of the components. The variables are described as follows:

- For each bid and offer block, the associated variable is the cleared quantity from that block.
- Generators and loads do not have any variable directly associated with them. The cleared quantities are the variables of the offer and bid blocks that are associate with the generators and loads. After the solve, post-processing will assign the sum of the cleared block quantities to the blocks parent, i.e., generator in the case of offer blocks, load in the case of bid blocks.
- Every bus has a phase angle variable associated with it. The solver adjusts the phase angle variable in order to allow branch flow to occur. The relationship between the phase angle differences, the branch susceptance and the branch flow, approximates the physical reality of the powerflow as described in Section 3.4. The phase angle of the reference bus is not a variable, it is fixed at zero.
- The variable directly associated with the branch is the branch flow.
- The branch has associated flow-loss segments. The variable associated with a flow-loss segment is the segment flow. The resulting loss does not need to be a separate variable; during the solve the product of the segment flow and the flow-loss ratio is assigned as a loss 50-50 to the buses at either end of the branch. In post-processing the loss on each segment is calculated as the product of the segment flow and its flow-loss ratio. The sum of these segment losses is assigned as the loss result for the branch.

Parameters:

a_{lp} = value of energy consumed by bid block p of load l

b_{lp} = maximum power that can be consumed by bid block p of load l

c_{mp} = cost of energy provided by offer block p of generator m

d_{mp} = maximum power that can be provided by offer block p of generator m

e_k = susceptance of branch k

f_k = max flow on branch k

g_{ks} = max flow on branch segment s of branch k

h_{ks} = ratio of loss to flow in branch segment s of branch k

Variables:

u_{ilp} = power cleared in bid block p of load l at bus i

v_{imp} = power cleared in offer block p of generator m at bus i

w_i = phase angle at bus i

x_{ijk} = power flow from bus i to bus j on branch k

y_{ijks} = power flow from bus i to bus j on branch segment s of branch k

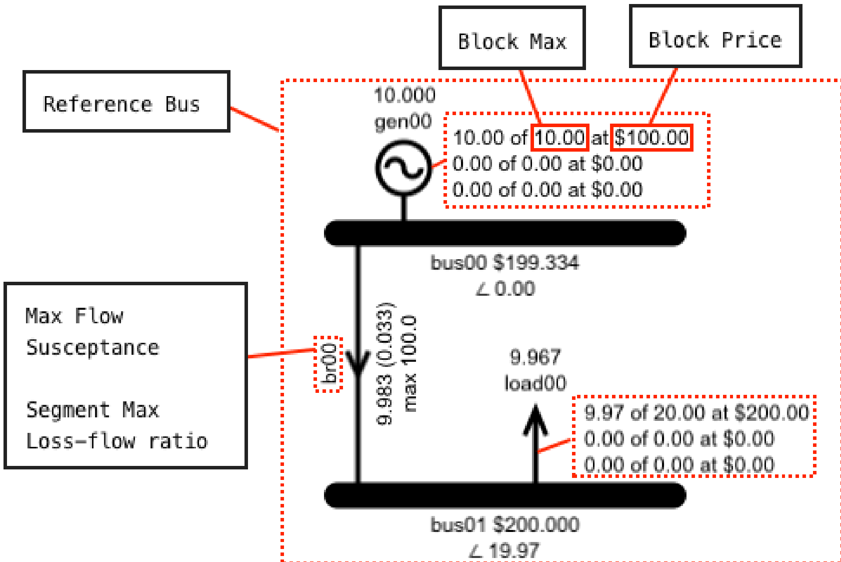


Figure 31: Parameters used by the LMP formulation.

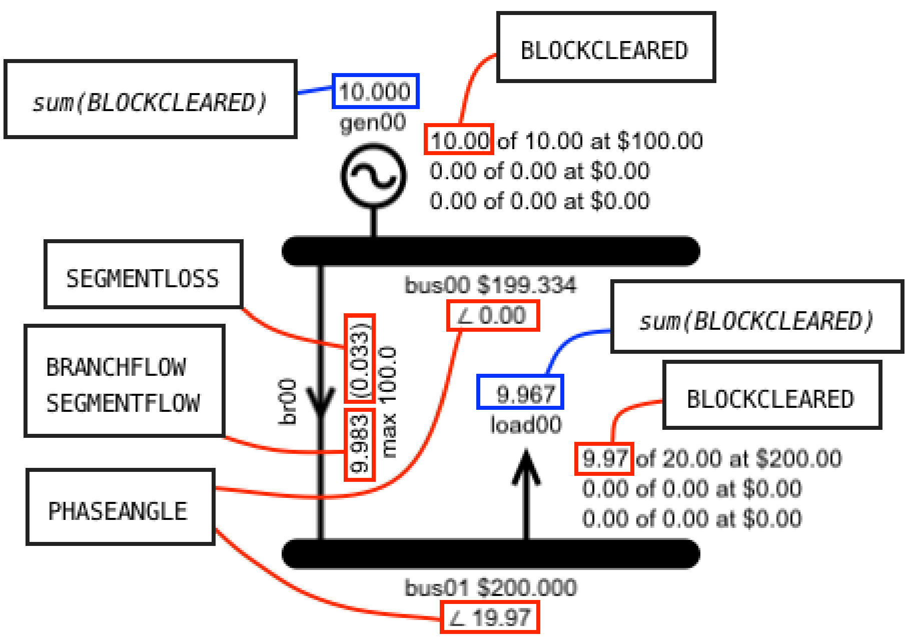


Figure 32: Variables used by the LMP formulation.

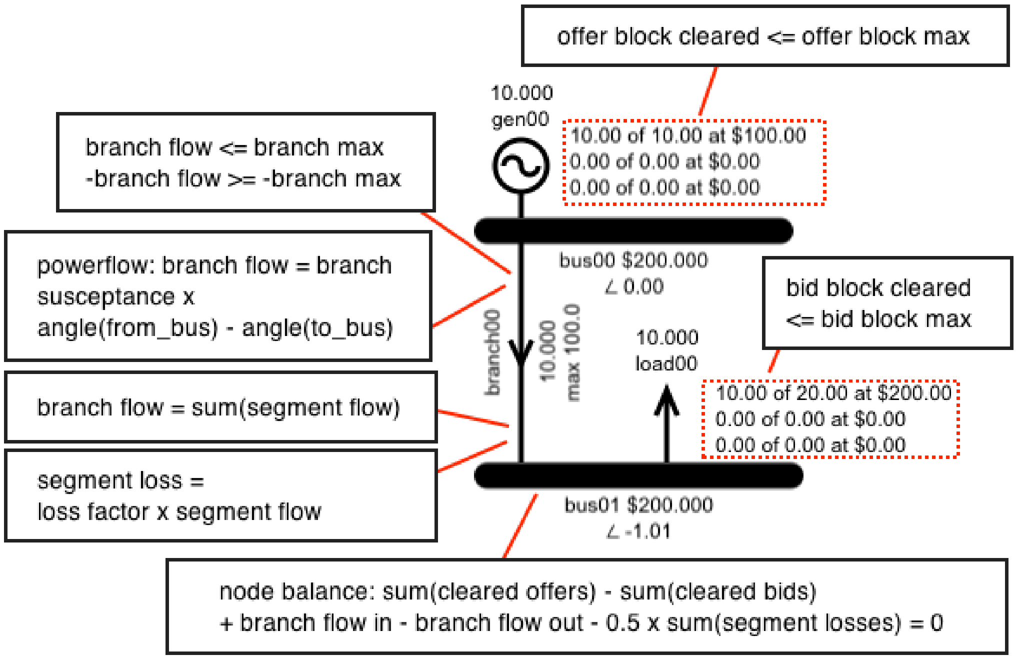


Figure 33: Constraints used by the LMP formulation.

Constraints. The objective of the LMP model is to maximise the value of the load that is provided, while minimizing the cost of the generation. The easiest way for the LMP model to maximise this objective would be to clear all the load bids without clearing any generation offers. However, this does not represent physical reality; the power that is provided must be produced by a generator and be transmitted to the load. The constraints within the LMP model enforce this physical reality. The constraints associated with the components in the model are shown in Figure 33 and are described as follows:

- The node balance equation enforces conservation of flow; the total power that flows into a bus must be equal to the power that flows out. The power that flows in is the sum of the cleared generation offers at the bus and the power that flows in via branches connected to the bus. The power that flows out is the sum of the cleared bids at the bus, the power that flows out via branches connected to the bus, and half of the losses associated with each branch that is connected to the bus, regardless of the direction of flow. The node balance is notable for the fact that it is the shadow price of this constraint that sets the buses price, i.e., the value to the objective that would result from an increase in the power available at the bus is determined by the value of relaxing node balance constraint.

- The simplified AC power flow equation described in Section 3.4 is modeled by the power flow constraint, which equates the flow on a branch to the product of the susceptance of the branch and the phase angle difference between the buses at either end of the branch.

- The simplified AC power flow equation relies on the phase differences between the buses to determine the power flow. In order for this to work properly the phase angle of the reference bus must be fixed at zero. In the LMP app this is not actually a constraint; the pre-processing for the solver simply excludes the phase angle of the reference bus from the solve by assigning it a coefficient of zero in all constraints.
- In order to ensure system security the flow on a branch is constrained to be less than or equal to the rating of the branch. Because branch flow can be in either direction there is a constraint to limit the flow in each direction.
- In order to model losses there is a constraint that enforces the branch flow to be the sum of the flows on the branch's associated flow-loss segments. The product of the flow on the flow-loss segment and its associated flow-loss ratio is then assigned as losses to the buses at either end of the branch, as described in the node balance constraint.
- Each of the flow-loss segments has a constraint that limits its flow to be less than or equal to the maximum flow of the segment.
- Each bid and offer block has a constraint that limits the amount cleared in the block to be less than or equal to the block maximum.

Unrestricted variables. In the physical world all the variables except the phase angle and the branch flow are restricted to positive values. In the LMP app the *unrestricted* phase angle and the branch flow variables are modelled using two positive variables, one of which is labelled as negative. In the constraints that use these variables both of the variables are included. When the pre-processing builds the constraint, the variable that is labelled as negative is assigned a negative coefficient in the constraint. In post-processing, the branch flow and the phase angle results are calculated by subtracting the result for the "negative" variable from the positive.

In the LP formulation laid out here, the branch flows are defined as directional, positive only variables, i.e., $flow_{ij}$ is positive and flow in the reverse direction, i.e., $flow_{ji}$, is also positive. The phase angle is defined as unrestricted.

Objective:

Maximise $$\sum_{l}^{Q}\sum_{p=1} a_{lp}u_{ilp} - \sum_{m}\sum_{p} c_{mp}v_{imp} \forall i \tag{48}$$

Subject to:

Bid block limit $$u_{ilp} \leq b_{lp} \qquad \forall i, l, p \tag{49}$$

Offer block limit $$v_{imp} \leq d_{ip} \qquad \forall i, l, p \tag{50}$$

Node balance $$\sum_{k}\sum_{j} x_{jik} - \sum_{k}\sum_{j} x_{ijk} - 0.5\sum_{k=1}^{q}\sum_{j=1}^{n} h_{ks}y_{ijks} + \sum_{m}\sum_{p} v_{imp} - \sum_{l}\sum_{p} u_{ilp} = 0 \qquad for all i \tag{51}$$

Reference bus $$w_i = 0 \qquad i = ReferenceBus \tag{52}$$

Power flow $$x_{ijk} = e_k(w_j - w_i) \qquad \forall k \tag{53}$$

Branch flow limit $$x_{ijk} \leq f_k \qquad \forall k \tag{54}$$

Branch segment flow $$x_{ijk} = \sum_{s} y_{ijks} \qquad \forall k \tag{55}$$

Branch segment limit $$y_{ks} \leq g_{ks} \qquad \forall k, s \tag{56}$$

Non-negativity $$u_{ip} \geq 0 \quad v_{ip} \geq 0 \quad x_{ijk} \geq 0 \quad y_{ijk} \geq 0 \tag{57}$$

The objective function (48) maximises the difference between the value of all the cleared bids and the cost of all the cleared offers. Constraint (49) limits the cleared quantity in each of the bid blocks to the maximum capacity of the block, while constraint (50) ensures that offer blocks are similarly limited. Constraint (51) is the node balance constraint which enforces that the power that flows into a bus, from branches or generators, is equal to the power that flows out, via branches or loads, or due to the losses on the branches. Half of a branch's losses are assigned to the bus at each end of that branch. Constraint (52) sets the phase angle of the reference bus to zero; all other phase angles are relative to this. Constraint (53) determines the power flow through a branch based on the susceptance of the branch and phase angle difference between the buses at either end of the branch. Hence, the phase angles are the variables that the solver adjusts in order to determine the power flows that maximise the objective. Constraint (54) limits the branch flow to the maximum flow of the branch. Losses are a consequence of branch flow and this is modelled by constraint (55) which enforces the relationship between a branches total flow and the flow on its flow loss segments, where each segment has a limit enforced by constraint (56).

4.4 Comparison with the minimum-cost network flow problem

The formulation for the minimum-cost network flow problem is as follows [2]:

$$\text{Minimize cost:} \qquad \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{58}$$

$$\text{Subject to:} \tag{59}$$

$$\text{Node balance} \qquad \sum_{j=1}^{n} x_{ji} - \sum_{j=1}^{n} x_{ij} = b_i \qquad i = 1, 2, ..., n \tag{60}$$

$$\text{Arc capacity} \qquad 0 \leq x_{ij} \leq f_{ij} \qquad j = 1, 2, ..., n \tag{61}$$

$$\tag{62}$$

The objective of the minimum-cost network flow problem is to minimize the cost of moving commodities through a network, subject to the constraint that the demand must be met and flow must be balanced. The LMP model formulation also has a constraint that flow must be balanced. However, LMP does not have a constraint that the demand must be met, rather the objective is to maximise the benefit of the demand that is met, where the benefit is the product of the quantity that is met and its value. The objective also includes a penalty that minimises the cost of the supply, where the cost is the product of the quantity that is supplied and its price. The other difference is that in the LMP model formulation the transport is determined by the power flow constraint and the associated losses. There is no constraint enforcing that power does flow, rather the phase angles that allow the powerflow are variables that the solver can adjust in order to allow demand to be met and therefore improve the objective value.

4.5 Summary

In this section we have listed all the components in the LP formulation of the LMP model and related them to the physical and economic reality that they represent. We then listed and explained the parameters, variables and constraints associated with the parameters. We then used these to present the LP formulation of the LMP model, followed by an explanation of each of the terms in the formulation. We concluded by comparing the LMP formulation with the formulation for the minimum cost network flow problem, highlighting what they have in common and also what makes the LMP formulation different from more generic minimum cost flow problems.

5 Tutorial guide for using the iPhone/iPad LMP app

The goal of this project is to explain the workings of LMP. Describing the LMP formulation is one step towards achieving this goal. The next step is to demonstrate LMP in a hands-on, interactive manner. This has been achieved by the author writing an application that provides portable, self contained access to an environment that can be used to build and solve LMP models. Appendix C provides some background on the writing of the application software. The application has been written to run on an iPad/iPhone because this provides an interface which naturally lends itself to the building and manipulation of graphical models. It is also a commonly available device and includes the facility to export reports by means of email. This section will demonstrate how the LMP app makes it easy to build, solve and learn from LMP models.

We first give an introduction to using the LMP app, including an overview of touch screen terminology, followed by a description of the controls specific to the LMP app. Figure 34 shows the app as it appears on the iPhone. The drawing area is sized for the iPad and while all of the drawing area is available of the iPhone, only part of it is visible in the display window; zoom and pan are used to access the complete drawing area. For the smaller models zoom and pan won't be needed but they are described later on in Section 5.5. The terminology used for interacting with the touch screen is explained in Table 2.

In the following sections a series of tutorials describe how to build, solve and analyse LMP models of increasing complexity. Each of these tutorials demonstrates some aspect of LMP, e.g., how the marginal price is set, or various phenomenon observed in the LMP results, e.g., "the spring washer effect".

5.1 Tutorial #1: Build and solve a one bus, one generator, one load model

Figure 35 shows the most simple model while still showing something interesting; generation is cleared to meet a load requirement and a bus price is set by the marginal component. The following steps describe how to build and solve this model and then the results are discussed. This will provide an introduction to the concept of marginal pricing. The components of the model are added to the LMP *model* by adding them to the *display*. A new component is added by tapping the toolbar button corresponding to that component. Due to the limited space on the iPhone display, on the iPhone the buttons that add the components are not on the main toolbar, they are on a separate toolbar accessed via the Build button Build. Now we are going to build the LMP model shown in Figure 35.

Turn panning off. If using the iPhone, *panning* allows models to be built that are larger than the iPhone display. However, with panning switched on, components can only be moved or re-sized by first tapping them to mark them as *selected*. Test users found this disruptive, so as we are only building small models to start with, we will turn panning off by tapping the panning button so that it toggles panning to off. Panning is discussed again when we load a large model in Section 5.5.

Add the first bus. If using the iPhone, tap the Build button Build to set the toolbar that adds components. Then tap the Bus button Bus to add a bus component. Because this is

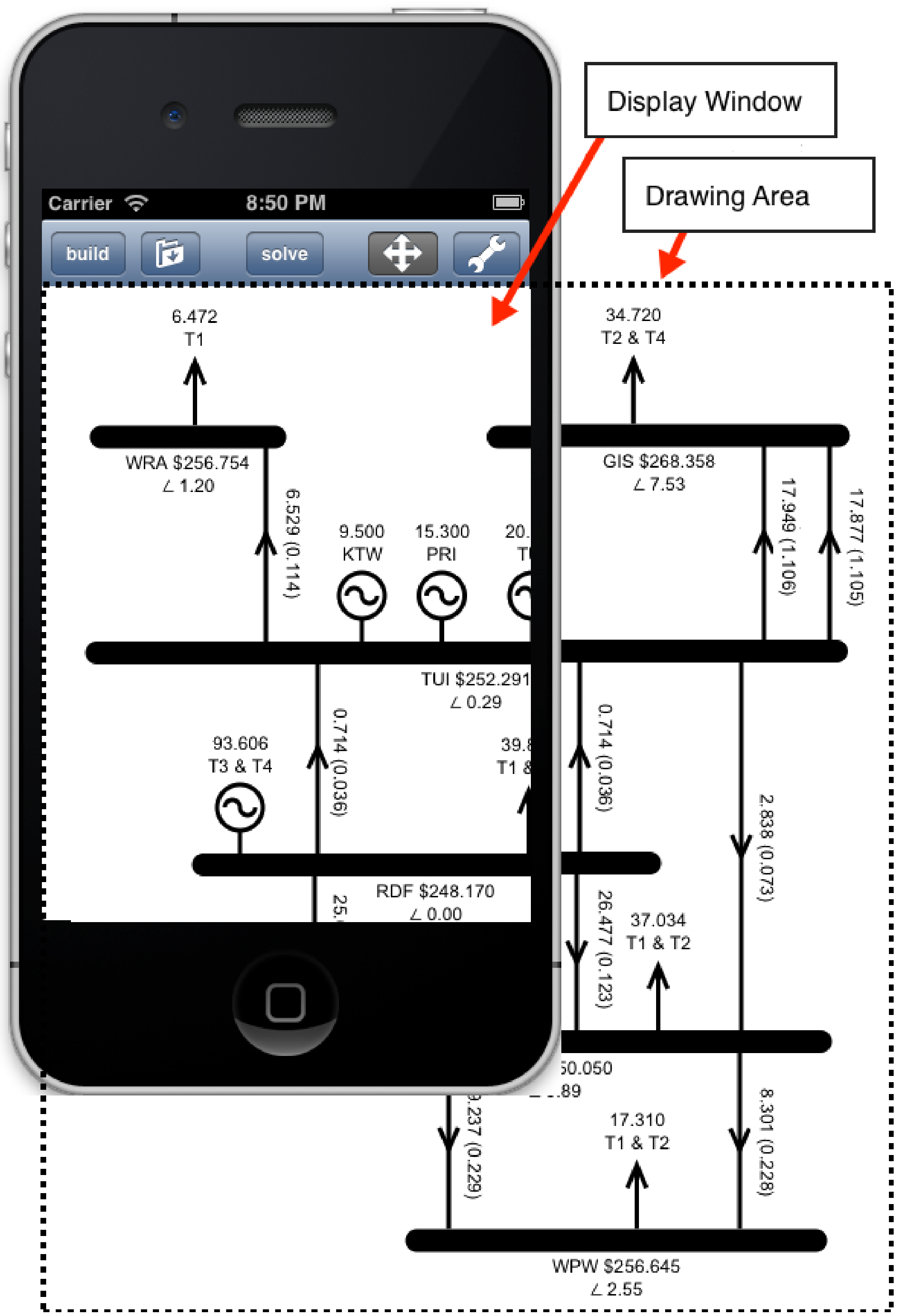


Figure 34: Display window and drawing area on the iPhone.

Interaction	Description
Touch	A physical touch to the screen with a finger or stylus. The display is not sensitive to how hard the touch is pressed but it is sensitive to how the touch moves. Once the screen is touched, the touch is said to *begin* and the software starts to track the movement of the touch.
Gesture	A touch that moves in a certain way over a certain time.
Touch Ended	Either the touch is physically removed from the screen, or the software ends the touch by deciding to ignore it. If the software ends the touch then the touch can be re-started by removing the physical touch and then initiating a new touch, e.g., when a component snap-connects to a bus, the software ends the touch.
Tap	A gesture. As the name suggests, the screen is touched briefly and then the touch is ended, e.g., tapping a button to initiate an action.
Long touch	A touch that is held without moving for a pre-defined length of time, e.g., to select a component for deletion requires a long touch of two seconds.
Pan	A gesture where the touch is held and moved. This is used to move objects around on the display, or to move the drawing area around within the display window.
Pinch zoom	A gesture where two fingers are used to pinch the screen; if the fingers are moving together then the image gets smaller, if the two fingers are moving apart then the object gets larger.
Swipe	A brief touch that starts and ends while moving, e.g., in the Load/Save data entry display this will select a row for deletion.

Table 2: Touch screen interactions.

the first bus added it will be the *reference bus*. When the model is solved, the reference bus will have a fixed phase angle of zero, see constraint (52) on page 46. Double tapping the bus will show its properties; the only bus property is whether or not it is the reference bus, as shown in Figure 36.

Add a generator. Tap the Gen button 

to add a generator . If it is already connected to the reference bus then it will be the same colour as the bus. If it is not the same colour as the bus then it will need to be moved in order to connect it to the bus.

The selected component. Only the selected component can be moved. When a component is added it will become the selected component, as indicated by a square edged border, as shown in Figure 35 where load00 is the selected component. A component can be set as the selected component by tapping it. The border around the selected component is also an indicator. The area inside the border indicates the touch area for the component. The colour of the border indicates the connection status of the component. Only fully connected components are included in the model; a generator or load is fully connected if it is connected to a bus, a branch is only fully connected if both ends are connected to a bus. When a component is fully connected the border will be green, otherwise it will be red.

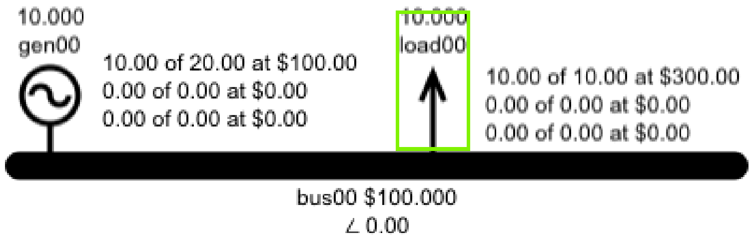


Figure 35: LMP result for simple one bus system, with generator setting the bus price.

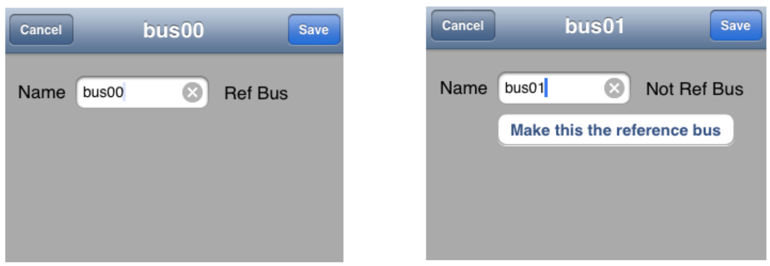


Figure 36: Example of the data entry screen for the bus component.

Adjusting a component. Adjusting a component refers to moving it or re-sizing it. Buses and branches can be re-sized or moved, generators and loads can only be moved. To demonstrate this, select the generator that you have just added and move it, by panning it as described in Table 2, so that it connects to or disconnects from the bus. Once a component is dragged close to a bus it will snap-connect, i.e., the component will be automatically adjusted to close the gap between the component's connection point and the bus. At the same time, the touch will be ended, i.e., the component can no longer be adjusted by the current touch. In order to adjust the component, a new touch must be initiated by lifting the finger and then touching the component again. Ending the touch in this way ensures that the position that the component snaps to is not accidentally adjusted.

When a component is connected to a bus, or disconnected, the software updates the LMP model to record the connectivity status. This connectivity is also represented visually; if a component has connectivity to the reference bus, either directly or via other components, then it will be the same colour as the reference bus. Otherwise it will be the disconnected colour.

How to delete a component. If you added a component by mistake you can delete it. To delete a component, touch the component and hold the touch. After two seconds the component will shake and a small cross will appear on the component, as shown in Figure 37. Tap the small cross to delete the component, or tap anywhere on the display background in order to cancel the delete.

Add a load. Tap the Ld button 

to add a load . Connect it to the bus, following the same procedure as was used for the generator.

Figure 37: Component selected for deletion.

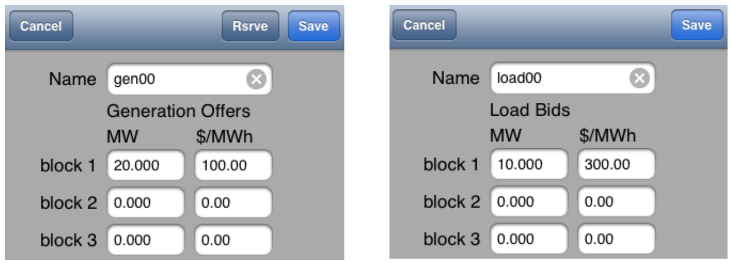


Figure 38: Data entry screens for (a) generator offers and (b) load bids.

Set generator and load properties. Set the properties of the generator by double tapping the generator to display its properties window. Enter the offer quantity and price for block 1 as shown in Figure 38(a). Set the properties of the load by double tapping the load and entering the bid quantity and price for block 1 as shown in Figure 38(b).

Solve the model. To solve the model, if on the iPhone return to the main toolbar by tapping the Done button 

, then tap the Solve button 

. The Solve button will be disabled ("greyed-out") while the solve is in progress; when the Solve button is enabled again the solve is complete.

Results. The expected result is shown in Figure 35. Above the generator is the total quantity cleared from the generator's offer blocks, i.e., 10MW. Above the load is the total cleared from the load's bid blocks which is also 10MW. Also shown are the details for the individual offer and bid blocks; the default option is that these details are not shown, in order to see them tap on the cleared total, i.e., the 10MW above the Gen or Load. This only shows the results for the Gen or Load result that was tapped, to show the details for all Gen and Load, go to the settings display by tapping the settings icon  and switch on the "Show Bids and Offers" option.

The marginal price at the bus is the shadow price of the corresponding node balance constraint, i.e., the increase in the objective value that would result from allowing the node balance constraint to be broken by 1MW by making another 1MW available at the bus. In Figure 35, the bus price is $100. This is because 1MW extra at the bus would allow 1MW less of generation to be cleared, which would decrease the costs, and hence increase the value, by:

$$1MW \times \$100 perMW = \$100$$

How the app produces the result. As the graphical model is built, the software records the components, their connectivity and their properties. When the Solve button is tapped, the software uses this data to create the simplex tableau for the LMP model, based on the variables and constraints of the LMP formulation that were described in Section 4. Figure 39 shows the initial tableau for the model we have just solved; the data for this figure was exported from the LMP app as a file of comma separated values, the only extra work required to produce this figure was to load the data into Excel [19].

The node balance constraint is an equality constraint, i.e., flow into the bus is equal to flow out. From the tableau we can see that this is modelled as a $\geq$ constraint and a $\leq$ constraint. The limits on the cleared quantities, i.e., the limits on the bid and offer blocks, are $\leq$ constraints. The generator and the load components which are the parents of these blocks do not need to be included; a post-processing procedure calculates the totals for the generator and load from their results of their blocks. Although the generator and load each have three offer or bid blocks, blocks with a maximum quantity of zero are excluded from the model by the pre-processing, as they serve no useful purpose.

The software uses an implementation of the simplex algorithm [18][11] (written by the author) to solve the LP model on the iPhone/iPad. The final tableau is shown in Figure 40. The column at the left hand side lists the constraints, along with which column number is basic for each constraint. The constraints which have basic variables that are not slack variables are:

- the node balance $\leq$ constraint, with a RHS of 10, which has column 0 as its basic variable, i.e., the cleared quantity for bid00; and
- the load block max constraint, with a RHS of 10, which has column 1 as its basic value, i.e., the cleared quantity for offer00.

From these results the post-processing extracts that both the bid block and the offer block cleared 10MW.

To work out the marginal price of a component we look to see what the reduced cost is for the slack variable of the constraint that limits the component, i.e., how much would the objective value change if the limiting constraint was relaxed. The marginal price of interest is the bus price because this is the price that generators will be paid and loads will be charged. The constraint that limits the bus is the node balance constraint. This is an equality constraint modelled as a $\leq$ and a $\geq$ constraint. Relaxing the $\leq$ constraint is the equivalent to adding \$0 generation at the bus, relaxing the $\geq$ constraint is the equivalent of adding \$0 load at the bus. From the final tableau, shown in Figure 40 we can see that the reduced cost for the node balance $\leq$ constraint for bus00 is 100, hence the post-processing sets the bus price at \$100 [20].

Other information in the tableau. The marginal prices of the bid and offer blocks are of no interest. But out of interest we can look at them in the tableau and explain them. From the tableau we can see that the marginal price for the offer block is zero, this is because more

[19] Currently the tableau can only be exported when running in debug mode, but it is planned to include the option of exporting the tableau from the LMP app via email.

[20] If it had been the $\geq$ constraint instead of the $\leq$ constraint that was non-zero then the bus price would have been negative. Negative bus prices are discussed below in Section 5.4 which covers the spring-washer effect.

Variables---> Constraints	RHS	load00_bid00 {BlockCleared}	gen00_offer00 {BlockCleared}	bus00 {PhaseAnglePos}	bus00 {PhaseAngleNeg}	bus00 {NodeBalanceLTE} Slack	bus00 {NodeBalanceGTE} Slack	load00_bid00 {BidBlockMaxLTE} Slack	gen00_offer00 {OfferBlockMaxLTE} Slack
bus00 {NodeBalanceLTE} (basic col: 4)	0	1	-1	0	0	1	0	0	0
bus00 {NodeBalanceGTE} (basic col: 5)	0	-1	1	0	0	0	1	0	0
load00_bid00 {BidBlockMaxLTE} (basic col: 6)	20	1	0	0	0	0	0	1	0
gen00_offer00 {OfferBlockMaxLTE} (basic col: 7)	10	0	1	0	0	0	0	0	1
ObjectiveFn (basic col: 0)	0	-200	100	0	0	0	0	0	0

Figure 39: Initial simplex tableau.

Variables-> Constraints	RHS	load00_bid00 {BlockCleared}	gen00_offer00 {BlockCleared}	bus00 {PhaseAnglePos}	bus00 {PhaseAngleNeg}	bus00 {NodeBalanceLTE} Slack	bus00 {NodeBalanceGTE} Slack	load00_bid00 {BidBlockMaxLTE} Slack	gen00_offer00 {OfferBlockMaxLTE} Slack
bus00 {NodeBalanceLTE} (basic col: 0):	10	1	0	0	0	0	0	1	0
bus00 {NodeBalanceGTE} (basic col: 5):	0	0	0	0	0	1	1	0	0
load00_bid00 {BidBlockMaxLTE} (basic col: 1)	10	0	1	0	0	-1	0	1	0
gen00_offer00 {OfferBlockMaxLTE} (basic col: 7)	10	0	0	0	0	1	0	-1	1
ObjectiveFn (basic col: 0)	2000	0	0	0	0	100	0	200	0

Figure 40: Final simplex tableau.

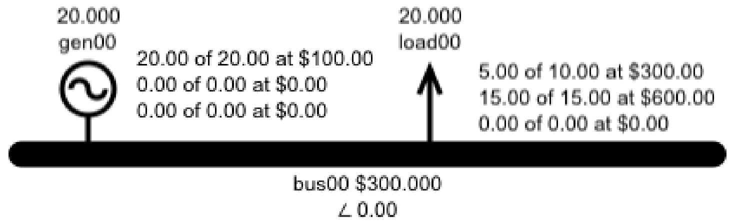
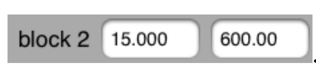


Figure 41: LMP result with the load setting the bus price.

generation is of no benefit; generation offers are a cost and can only provide a benefit if they allow bids of a higher value to be cleared. All the bids are already fully cleared, so the generation offer can provide no benefit. The marginal price for the bid block is $200 because if the limit on the block quantity was broken it would allow another 1MW of the $200/MWh bid block to clear, giving a benefit of $200.

The marginal component. In the example shown in Figure 35, because the load bid is fully cleared, it is the generation that sets the price. If the node balance constraint at the bus was relaxed to effectively make another 1MW of generation available at the bus then the objective value would improve because 1MW less of actual generation would need to be cleared. Because the generation costs $100/MWh the increase in the objective value would be $100. The generator at the bus is the *marginal plant*, i.e., it is the generator that is setting the marginal price, i.e., the shadow price, at the bus. This is the price that the generator will be paid for each MWh provided and the load will pay for each MWh consumed. The price also indicates that extra generation at this bus can expect to be paid $100, i.e., no more than the existing generation because the existing generation still has spare capacity.

Editing the model to change the marginal component. Having a generator as the marginal plant is the normal course of events in the actual electricity market. The load bid is fully cleared, which means that all the load is supplied. If the load bid were not fully cleared then this would indicate that load would need to be reduced, i.e., someone would not be supplied with electricity. This would also mean that the load would be the marginal plant and the price of the uncleared load bid would set the marginal price. We can demonstrate this by adding another load bid:

- Double tap the load to edit the load bids.
- Add block 2 data of 15MW at $600/MWh, i.e., 

.
- Tap the Save button.
- Tap the Solve button.

The result is shown in Figure 41. Block 2 of the load fully clears because at $600/MWh it adds the most to the objective value. Block 1 of the load only clears 5MW, i.e., it does not fully clear because there is not enough generation. In this case making an extra 1 MW available at the bus would allow 1MW to be cleared from load block 1, which would increase

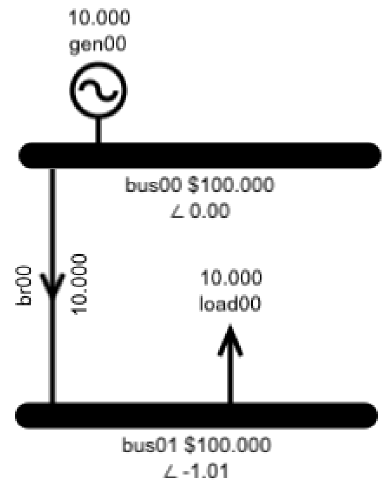


Figure 42: LMP result with branch added (lossless).

the objective value by $300. Hence the load sets the bus price at $300. This indicates that the load was not fully cleared, in this case there was 5MW of load that could not be supplied and would need to be switched off, i.e., there would need to be 5MW of *load shedding*. This price also indicates that, on this occasion, extra generation at the bus can expect to be paid $300. The bus prices, taken over time, indicate whether or not it is worth building new generation.

Before we continue, we will remove the block 2 bid from the load, so that we are solving a system that does not have load shedding. To remove bid block 2 from the model, edit the properties of the load and enter 0MW and $0/MWh for bid block 2.

5.2 Tutorial #2: Adding a branch

We are going to extend the single bus model created in Tutorial #1, by adding a branch and another bus, so that it looks like the network diagram shown in Figure 42. This will demonstrate what happens when generation and load are at different locations, as is the case in the real world.

- Starting with the simple system we have just built, add a branch by tapping the Branch button 

. Branch and bus components can be moved *or* re-sized; a touch in the centre third of the component initiates a move, a touch on the end thirds initiates a re-size, as shown in Figure 43. If the branch that was added is not connected to the bus, bring its end to the bus, by moving or re-sizing. Once the end of the branch gets close to the bus it will automatically snap-connect.

- Double tap the branch to display the data entry screen. Enter the branch parameters as shown in Figure 44. The characteristics of the branch are its resistance R and its reactance X. As described in Section 3.4, the model formulation uses the susceptance of the branch in the simplified powerflow calculation. The susceptance is calculated from the resistance and the reactance, so entering resistance and reactance provides all the necessary information for the software to calculate the susceptance.

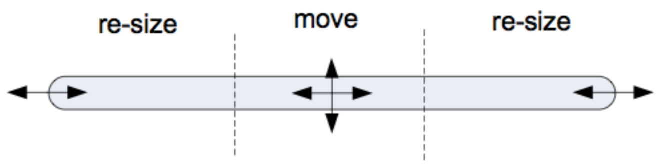


Figure 43: Branch and bus adjustments: Touch at the ends to resize, touch in the middle to move.

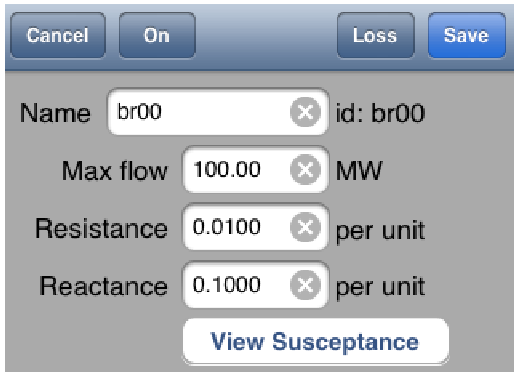


Figure 44: Data entry screens for branch.

However, we may want to replicate the result of an actual market model (see Section 5.5 below). The data for the New Zealand electricity market model is provided as resistance and susceptance, so in that case it would be easier if we could enter the susceptance, rather than the reactance. Hence the data entry form also has the option of entering the susceptance directly; tapping the "View Susceptance" button will show the susceptance value calculated from the resistance and reactance, but will also allow it to be entered directly.

- Add a new bus by tapping the Bus button . If necessary move it so that it snaps to the other end of the branch.

- Move the load that is currently connected to bus00 by panning it down so that it snaps to the bus we have just added. Generation and load cannot be re-sized, only moved, so it does not matter where the load is touched in order to move it.

- Tap the Solve button.

The result should be as shown in Figure 42. The offer block clears at bus00 and flows down the branch to the load at bus01. Due to the power flow constraint, in order for the branch flow to occur the solver has had to adjust the phase angle at bus01. The phase angle at bus00 is fixed at zero because it is the reference bus.

5.2.1 Solving with branch losses

By default branch losses (as described in Section 3.5) are not included in the model. This makes it easier to demonstrate the basic concepts of marginal pricing and power flow. With

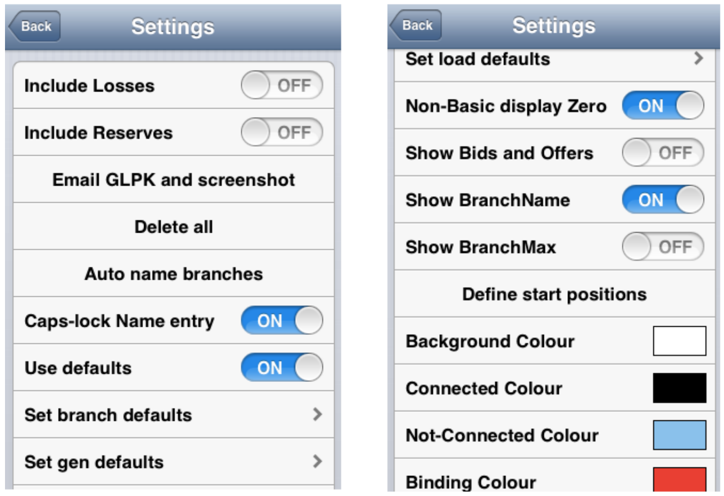


Figure 45: The settings display.

these explained, now we want to see what happens to the result when branch losses are included.

Settings. The branch losses option is selected via the settings display, which is accessed by touching the Settings icon  on the main toolbar. Figure 45 shows the settings display, which is used to alter such things as:

- The way that the data is entered, e.g., default values, caps lock.
- The way that the optimisation problem is solved, e.g., include losses.
- The way that the components are displayed, e.g., the colour of components connected to the reference bus and not connected to the reference bus can be set here.
- Other, e.g., exporting data, deleting all data.

Including branch losses. To include branch losses in the model, go the the settings display and tap the switch next to the "Include Losses" label, so that the switch status changes from Off to On, i.e., 

. Then tap the Solve button. The expected result is displayed in Figure 46. This shows that there are losses on the branch of 0.033MW. The losses on the branch are proportional to the flow on the branch. When the maximum flow value for the branch is saved, the software calculates the flow loss tranches using an algorithm that divides the branch flow in three and calculates the loss at each break point. The flow-loss tranches can be viewed by double tapping the branch to view its properties display then tapping the "Loss" button, as shown in Figure 47. The flow-loss tranches can also be edited, which will be useful when modelling a real system that uses a different algorithm, such as when we model part of the New Zealand electricity market system in Section 5.5.

In the model used by the LMP app, half of the losses are assigned to the buses at each end of the branch, i.e., in this result the total losses are 0.033MW, half of these are assigned

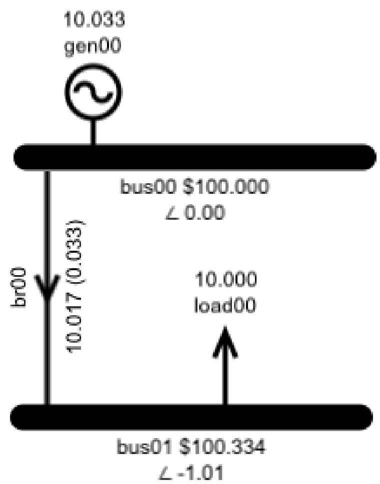


Figure 46: LMP result with branch losses.

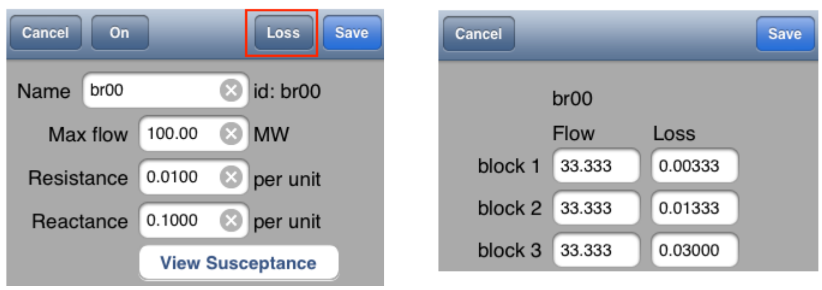


Figure 47: Viewing the branch flow-loss tranches.

to bus00, hence the flow out of bus00 is, to 3 d.p., 10.033MW $-\frac{0.033MW}{2}$ = 10.017MW. This is the flow on the branch. The other half of the 0.033MW losses are assigned to bus01, hence the power available to the load is 10MW.

Prices in a system with branch losses. In the system with branch losses, the shadow price at the load bus is higher than the generator bus because relaxing the node balance constraint at the load bus by 1MW allows an objective value improvement equal to reducing the generation cost by *more* than 1MW. This is because the resulting generation decrease would be the 1MW supplied to the load *plus* the 0.033MW supplied to the losses. This is more than the improvement arising from relaxing the node balance constraint by 1MW at the generator bus, which only allows a generation decrease of 1MW exactly.

Comparing the results with GLPK. The result shown in Figure 46 includes all the LMP model features currently implemented by the LMP app: bus, branch, gen, load, offers, bids and branch losses. Now is a good opportunity to confirm the results produced by the LMP app by loading the LMP model into the GNU Linear Programming Kit (GLPK) software, solving the model and comparing the results [72]. GLPK does not run on iOS, but the LMP application has been coded so that it can export the current LMP model in the GLPK mod-

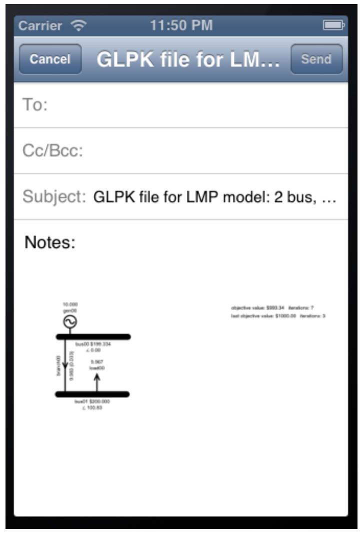


Figure 48: Email ready to send screenshot and GLPK model file.

elling language and email it to somewhere that GLPK can be run. To export the LMP model, from the Settings display, tap the "Email GLPK and screenshot" option. This will create an email as shown in Figure 48. The email includes the GLPK model and a screenshot of the model. Enter the recipient's email address and then tap the Send button Send. GLPK can be downloaded free from `http://www.gnu.org/software/glpk/`.

The GLPK model created by the LMP app and the results produced by GLPK are shown in Appendix B. By comparing the Activity and Marginal columns in the GLPK results with the results shown in Figure 46 it can be confirmed that the quantities and prices produced by GLPK are the same as those produced by the LMP app. This is an important validation of the simplex solver implemented in the LMP app.

5.2.2 Binding branch

In a model that includes branch losses, the losses will always have an impact on bus prices. Now we will look at the impact on bus prices when a branch reaches its limit, i.e., when a branch is *binding*. A binding branch will have more of an impact on prices than losses, but normally binding branches do not occur in every result. We will now modify our network diagram so that we have a binding branch.

- To keep things simple we will turn off losses for now, by going to the Settings display and selecting "Include Losses" to OFF.
- Double tap on the branch and adjust the maximum flow to be 9MW. From the previous result (with no losses, see Figure 42) we know that the solver would like to send 10MW through the branch.

When we solve, we observe that the transfer from the load to the generator is restricted by maximum of the branch, as shown in Figure 49. Because the branch is a *binding* constraint it has been coloured red.

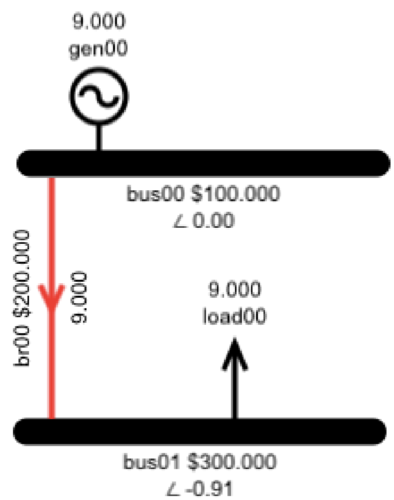


Figure 49: Binding branch causing price separation.

The shadow price at the load bus is now set by the load bid price because not all of the load is met. The benefit of an extra 1MW at the load bus would allow another 1MW of the $300 bid to clear. The price at the generation bus is lower, set by the generation price, because an extra 1MW at that bus would not be able to reach the load, hence its only benefit would be to replace 1MW of the $100/MWh generation, hence benefiting the objective by $100. This result demonstrates a *binding* branch constraint (a branch constraint at its limit) leading to *price separation* (prices on one side of the branch are separated from the prices on the other side of the branch; each bus has no direct influence on the price at the bus at the other end of the branch).

In this case the branch's maximum flow constraint also has an associated price, namely the $200 that the objective would benefit by if the branch constraint was relaxed by 1MW to allow 1MW of the $100 offer to be used to clear the $300 bid.

5.3 Tutorial #3: Investigating the Power Flow constraint

So far in Tutorials #1 and #2 we have seen how either the generation or the load can set the marginal price. We have also seen the impact of a *binding* branch, i.e., a model where the branch flow is constrained by its maximum flow constraint. The binding branch constraint separates the prices at the buses on either side. By virtue of having solved a model with a branch in it we have seen the effect of the power flow constraint that we described in detail in Section 3.4; in to send power from one bus to the other it is compelled to use the branch's power flow constraint:

$$BranchFlow = BranchSusceptance \times \Delta BusPhaseAngles \tag{63}$$

In the single branch example, the bus phase angle variables have no impact other than enabling the branch flow to transport the generation to the load. As soon as we have more than one branch we can see some more interesting implications of the power flow constraint and the associated phase angles.

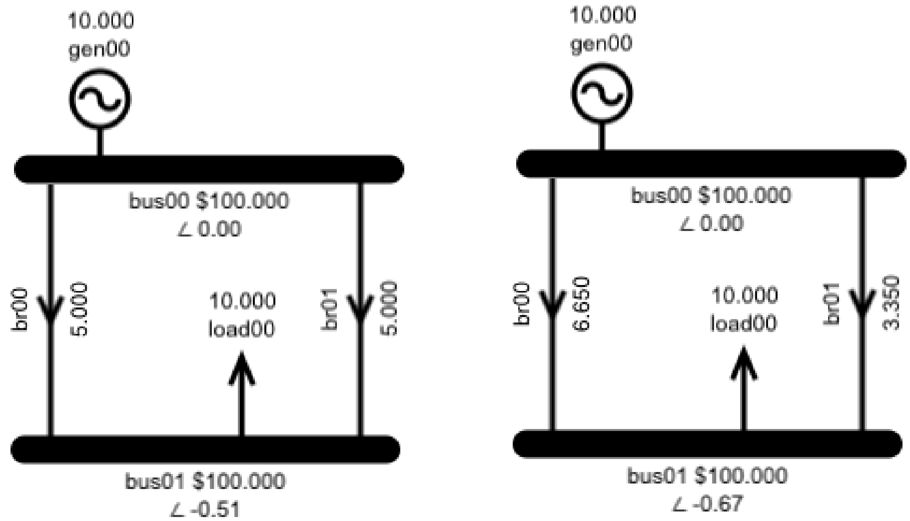


Figure 50: Parallel branches with (a) same properties (b) br00 has lower impedance.

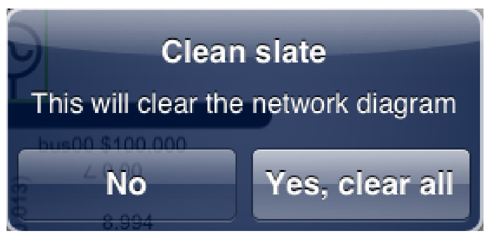


Figure 51: Prompt to warn that all components will be deleted, in response to selecting the "Delete all" option from the Settings display.

5.3.1 Parallel branches with the same properties

We are now going to build a model with parallel branches, as shown in Figure 50. To do this we can start with the model that we built in Tutorial #2 and then add a branch as shown in Figure 50, assigning the same properties as the first branch, i.e., as shown in Figure 44. Alternatively you can build the model from scratch, adding and connecting the components as shown in in Figure 50 and assigning the properties shown in Figures 38 and 44.

Deleting everything. If you choose the option of starting from scratch then all components of the existing model can be deleted by going to the Settings display and then tapping the "Delete all" button. You will be warned by the prompt shown in Figure 51 that everything will be deleted; tap "Yes, clear all" to continue or "No" to cancel the deletion.

Identical parallel branches. With the model built and solved, the expected result is shown in Figure 50 (a). The impact of the power flow equation is to force the flow to be the same on both branches; because they both see the same phase angle difference and they both have the same susceptance, the power flow constraint forces them to have the same flow.

5.3.2 Parallel branches with different susceptance values

In a real power system there are many parallel branches and they are not always identical. We will adjust our model to see the implications of parallel branches that are not identical and then we will discuss the implications that this has for a real power system.

Because the branch flow is determined by the product of the branch susceptance and the phase angle difference across the branch, branches that are connected between the same buses (and therefore are subject to the the same angle difference) that have different susceptance values must have different branch flows.

The susceptance is calculated from the reactance and the resistance, as described by Equation (20) on page 32. The susceptance is a slightly unwieldy number, the reactance is an easier number to update (and a direct property of the branch), so the value we will update is the reactance.

To update the reactance for br01:

- Double-tap br01 to show the properties display.
- If the Susceptance input is displayed, tap on the "Enter as Reactance" button.
- Change the Reactance from 0.1 to 0.2.
- To see what the Susceptance has changed to, tap on the "View Susceptance" button.
- Tap on the Save button.
- Tap on the Solve button.

The result is shown in Figure 50 (b). Because reactance is the property that opposes AC power flow, as expected increasing the reactance has decreased the flow on br01 from 5.0MW to 3.35MW. Notice increased flow on br00 also.

5.3.3 Binding parallel branches

In the previous example we demonstrated that parallel branches with different susceptance values will have different flows. This can cause a problem in the power system when one of these branches reaches its limit (becomes *binding*) while the other does not. Starting with the model from the previous example we will now demonstrate what happens when one of these parallel lines is binding and the other is not:

- Edit the property of br00 and set its maximum flow to 6.0MW. In the previous result it was *scheduled* 6.65MW.
- Edit the properties of br01 and set its maximum flow to 11.0MW.
- Tap on the Solve button.

The result is shown in Figure 52 (a). As expected the flow on br00 is now binding at 6MW. What has also happened is that the flow on br01 has reduced from 3.35MW to 3.02MW. Rather than increase its flow to take up the reduced flow on br00, br01 has actually reduced its own flow. This is because when the solver adjusts the phase angles on the buses in order to enable flow on the branches, it can only adjust the phase angles up to the point where

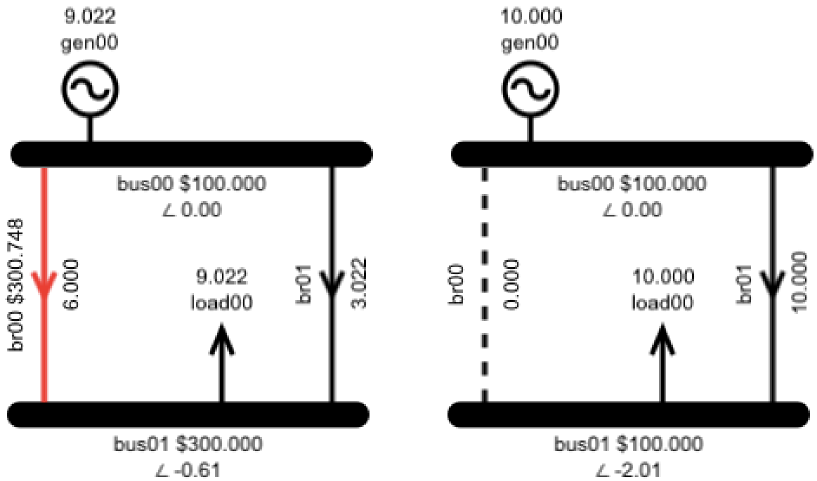
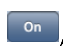
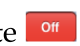


Figure 52: Parallel branches (a) flow restricted (b) restriction removed.

br00 binds. After that any increase in the phase angle difference would require br00 flow to increase, which cannot happen due to the maximum flow constraint. The br00 maximum flow limit constrains the phase angle difference and this in turn constrains the flow on br01.

Removing the binding branch from the model can improve the financial result. We will demonstrate this by removing br00, the binding branch. To remove br00 from the model without actually deleting it:

- Double-tap br00 to view its properties.
- In the toolbar, tap the On button 

, this will toggle the button to the Off state 

and remove the branch from service.
- Tap the Done button.
- Tap the Solve button.

Figure 52 (b) shows the result. Now br00 appears as - - - - - - to show that it is not included in the model. With br00 removed there is no longer any binding branch constraint and the solver can adjust the phase angle at bus01 in order to allow br01 to increase to its maximum flow. The load is now fully met and the bus prices return to $100/MWh. Note that this is not a better result in terms of system security; previously the system could stand to lose one of the branches and still supply some or all of the load, whereas after removing br00, the accidental loss of br01 would result in none of the load being supplied at bus01, which is known as a *loss of supply*.

Before continuing, return the values to their default settings:

- Set br01 reactance back to 0.1.
- Set maximum flow for both branches back to 100.

5.3.4 Parallel branches in the real power system

Parallel transmission circuits allow more power to be transmitted. The first reason for this is that there is a limit on the size of the wires carried by the pylons; once the transmission requirement gets beyond a certain level the only way to add more capacity is to add a parallel circuit. The other reason for parallel circuits is the requirement for the System Operator to ensure that the system is *secure*, which was discussed in Section 3.3. To recap; the System Operator needs to ensure that the loss of a single component will not result in a *loss of supply*. If all the power to a bus was carried on one large transmission circuit then if this circuit *tripped* (was removed from service due to a fault) then there would be loss of supply. If the power is shared with a parallel circuit then there will be less power on each circuit and it is possible to manage the system so that a single tripping does not result in a loss of supply.

An example of parallel branches being used to increase capacity is the case of South Island generation being transported to the upper North Island. Power is transported between the South Island and the North Island by the HVDC inter-island link [21]. When the South Island has plentiful generation, the power is sent north, entering the North Island at Haywards substation near Wellington. From there the power can travel further north. As the amount of transfer heading north has increased over the last 30 years or so, due to more capacity on the inter-island link and load growth in Auckland, more transmission circuits were added heading north from Wellington. These circuits were in parallel with existing circuits heading north. The new circuits had a higher capacity than the existing circuits, but because they were effectively in parallel with the existing circuits there were cases where the situation shown in Figure 52 (a) occurred, i.e., the transfer on the larger capacity circuits was limited by the smaller circuits in parallel.

In order to allow the new circuits to run to their full capacity, some of the existing circuits were removed from service, in order to achieve, in essence, the effect demonstrated in Figure 52 (b). Physically the transmission circuit is still in place, but a very large switch has been opened at one end to prevent the power from flowing. These breaks in the flow of transmission are referred to as *system splits*. System splits in the New Zealand system are described in detail on the System Operator website [45]. The system splits are included in the LMP model.

5.4 Tutorial #4: The Spring Washer effect

We have seen how the combination of the power flow constraints and a binding line can prevent a parallel line from being scheduled to its limit, even though this results in load not being supplied. Because supplying load is the only way for the objective value to increase, the solver will make every effort to make sure that load is supplied. Now we look at the "spring washer effect" [59][61], a case where the efforts that the solver makes in order to provide load can result in unusual pricing situations.

The New Zealand Electricity Authority website contains submissions from generators regarding the occasions when the spring washer effect has occurred [24][15] as well as a re-

[21] Also known as the *Cook Strait cable*. This is actually three cables. The AC power is converted to High Voltage DC (HVDC) before being sent along the cables and converted back to AC at the other end. Although the conversion equipment is expensive, HVDC has lower losses when transporting electricity over very long distances and it allows the power flow to be controlled exactly; the AC systems at either end can be run independently with the HVDC effectively acting as a load in one island and a generator in the other [1][22][73]

Figure 53: An actual spring washer; a separation with one side higher than the other, joined by a sloping continuum.

port on the spring washer effect from Dr Grant Read [59], while the System Operator website includes an animated explanation [46]. The spring washer effect preceeds the commercial use of LMP, with a thorough (and possibly the first) explanation of it appearing in Brendan Ring's 1995 Doctoral Thesis on the proposed New Zealand electricity market [61].

In the spring washer effect there is a binding branch and a parallel path. So far this is the same as the example that we looked at in Tutoral #3. The difference with the spring washer effect is that the parallel path consists of more than one bus. The resulting prices are as follows:

- There is price separation across the binding branch. We saw this in Tutorial #3; the bus price on one side of the branch has no direct influence on the bus price at the other side. This is because relaxing the node balance constraint on one side of the branch cannot result in any power flowing to the other side, due to the fact that the branch is at its limit.
- The price on one side of the binding branch is very high. This will be the side with un-cleared bids.
- The price on the other side of the binding branch will be the lowest price in the system.
- The prices at the buses on the parallel path will form a sloping continuum from the low price to the high price.

This description is very similar to the description of an item of hardware known as a spring washer, an example of which is shown in Figure 53. The spring washer has its highest point and lowest point separated by a break, while in the other direction they are physically connected via a sloping circuitous path.

The spring washer effect does not occur very often, but when it does happen it causes concern because it results in a very wide range of prices, with the high price being potentially thousands of dollars and the low price potentially being negative. A negative price means that a generator would have to pay in order to generate, or a load would be paid for consuming electricity.

A three bus model. All our models so far have only had two buses. To demonstrate the spring washer effect we will need at least three buses. Hence, to demonstrate and explain the spring washer effect, first build and solve the three bus model shown in Figure 54(a), using the default values for all components, i.e., the values shown in Figures 38 and 44.

In this model the power flow constraint causes the flow from bus00 to bus02 to be $\frac{2}{3}$ along br00 and $\frac{1}{3}$ along br01 and br02. The power flow is uneven because while the branches all

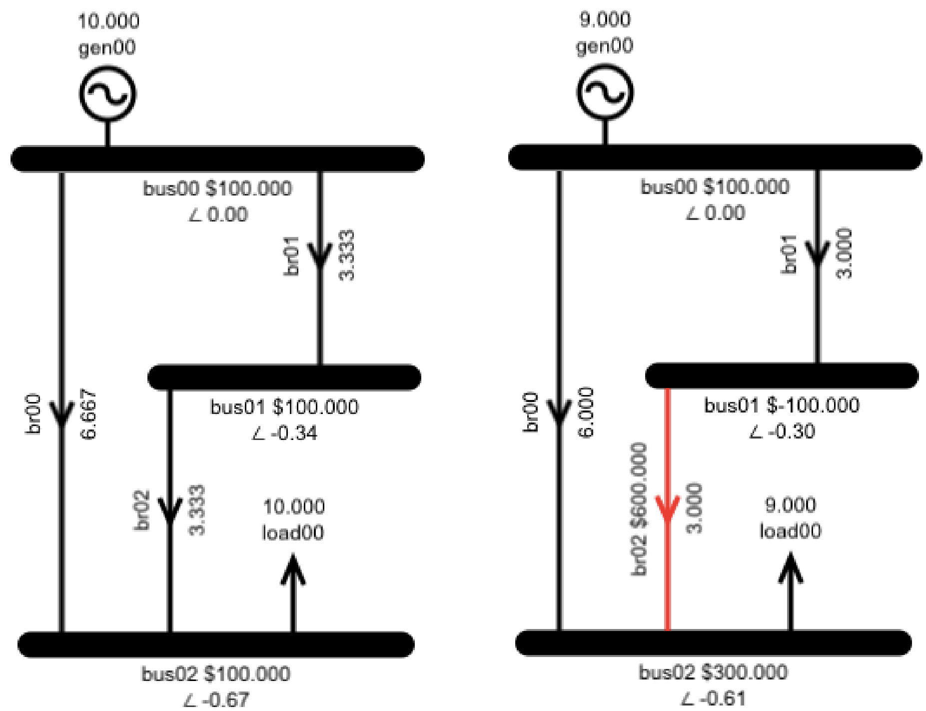


Figure 54: Three bus model (a) Unconstrained (b) Spring washer causing negative prices.

have the same susceptance the available paths between the generation and load do not. The flow along br01 cannot be the same as br00 because this would require the angle difference to be the same across both branches, i.e., the angle at bus01 would need to be the same as the angle at bus02, but this would allow no flow along br02 because there would be no angle difference across it. The optimal solution is that the flow is proportioned as shown in the result, which also reflects the actual physical power flow on such a system.

The three bus model with a binding constraint: spring washer pricing. When uneven parallel flows such as those in the three bus model combine with a binding constraint, the solver is still able to schedule flow along the path parallel to the binding constraint. This is because the ratio of the parallel flow to the binding flow is not one to one and this is what allows the spring washer effect to occur.

To demonstrate the spring washer effect we will cause a binding constraint on br02, currently scheduled 3.333WM, by editing its properties to lower its maximum flow limit to 3MW. This give the result shown in Figure 54(b), which demonstrates the spring washer effect. There is a continuum of prices that range from the high price on one side of the constraint to the low price on the other side. In this case the continuum is $300, $100, −$100. The shape of the spring washer is suggested by the fact that there is a split and between the highest and lowest prices, but they are joined by a sloping price path that makes its way around the split.

The negative price at bus01 represents the improvement to the objective function that would result from another MW of power being made available at bus01, i.e., the result is indicating that generating 1MW at bus01 would make the objective value $100 *worse*, as op-

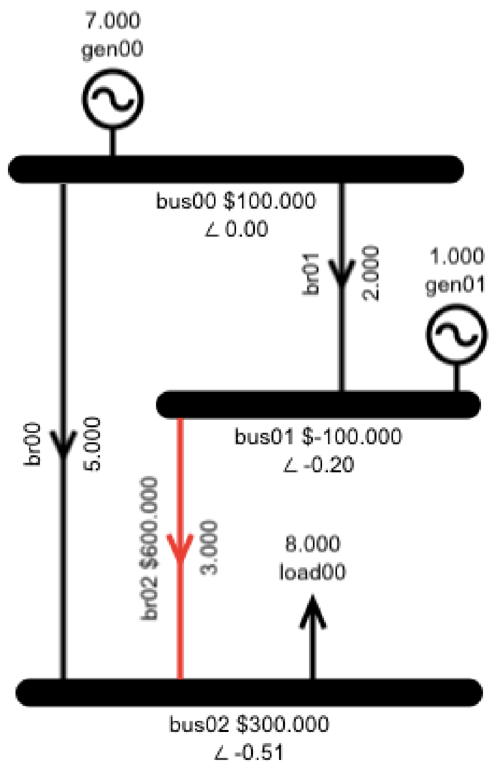


Figure 55: Extra generation results in worse objective value.

posed to the normal expectation that extra zero priced generation would make the objective value better.

5.4.1 Explaining negative prices

Explaining the negative price by adding generation. We can use the LMP app to explain the spring washer pricing in terms of the LMP formulation. In the spring washer case we have just created, shown in Figure 54(b), the objective value is:

$$objective = \sum_{Bids} Bid_{price} \times Bid_{cleared} - \sum_{Offers} Offer_{price} \times Offer_{cleared} \tag{64}$$

$$\Rightarrow objective = \$300 \times 9 - \$100 \times 9 = \$1800 \tag{65}$$

To see what would actually happen if an additional 1MW of generation was made available at bus01, we can add a generator to bus01, with a 1MW offer. However, even if we give this 1MW a $0 price, it will not clear. In order for the offer to clear, its price must be less than or equal to the value of new generation at that location, i.e., the shadow price of the bus, also referred to as the marginal price. If new generation has an offer *price* higher than the *value* of generation at that location then the solution will not clear the offer.

We know that the value of generation at bus01 is −$100, because this is the marginal price at that bus. Hence, we will:

- Add a new generator and connect it to bus01.

- Edit its properties to give offer block 1 a quantity of 1MW and a price of $-\$100/\text{MWh}$.
- Tap on the Solve button.

The result is shown in Figure 55. Because there is no load at bus01, the only way the generation can travel to the load is via br02. It cannot travel back to bus00 on br01 because this would require phase angle differences that would also require the flow on br00 to be reversed, which would result in no power traveling to the load. The only option is that the 1MW at bus01 travels to bus02 via br02. This requires that 1MW less travels from bus00 to bus01. The overall effect is that the phase angle difference between bus00 and bus02 now only allows for 5MW of flow, which effectively restricts br00 to 5MW of flow.

To see what impact the extra generation at bus01 would have on the objective value we calculate the objective for the result in Figure 55. If we include the new generation in the calculation we find that the objective value hasn't changed, which is as expected because we priced it exactly the same as the bus price in order for it to clear:

$$objective = \$300 \times 8 - \$100 \times 7 - \$ - 100 \times 1 = \$1800 \tag{66}$$

If we calculate the objective value without including the new generation, to see its effect, we get:

$$objective = \$300 \times 8 - \$100 \times 7 = \$1700 \tag{67}$$

This objective value is \$100 worse than the objective value before the 1MW of generation was added at bus01. This is why the value of generation at bus01 is $-\$100$; any generator at that bus would have to *pay* \$100 for every MW that they generated. And this is why if we make the value of the offer $-\$100$ then the offer will clear, because the solution is no worse than if it did not clear. If the offer price was $> -\$100$ then the offer would not clear, if it was $< -\$100$ then the offer would clear and the objective value would be better than the solution with no generation at bus01. At $-\$100$ the solver could choose to clear it or not (in our case it has cleared) [22].

Explaining the negative price by adding load. We have demonstrated that the price at bus01 is $-\$100$ because the objective value would decrease by \$100 if 1MW of extra generation was made available at bus01. Now we will use the LMP app to see what would happen if 1MW of extra load was added at bus01:

- Edit the model to delete the generation that was added at bus01. How to delete a component is described on page 52.
- Add a new load and connect it to bus01.
- Edit the properties of the new load to give it a quantity of 1MW and a price of \$0.

After solving, the result is shown in Figure 56(a). With load added at bus01, the flow on br01 can increase, which allows the angle at bus01 to increase. This in turn forces the angle

[22] This is effectively a case of multiple optimal solutions. In the actual market system it is unlikely that the model will be presented with situations like this because due to branch losses the energy prices at the bus are unlikely to be round numbers.

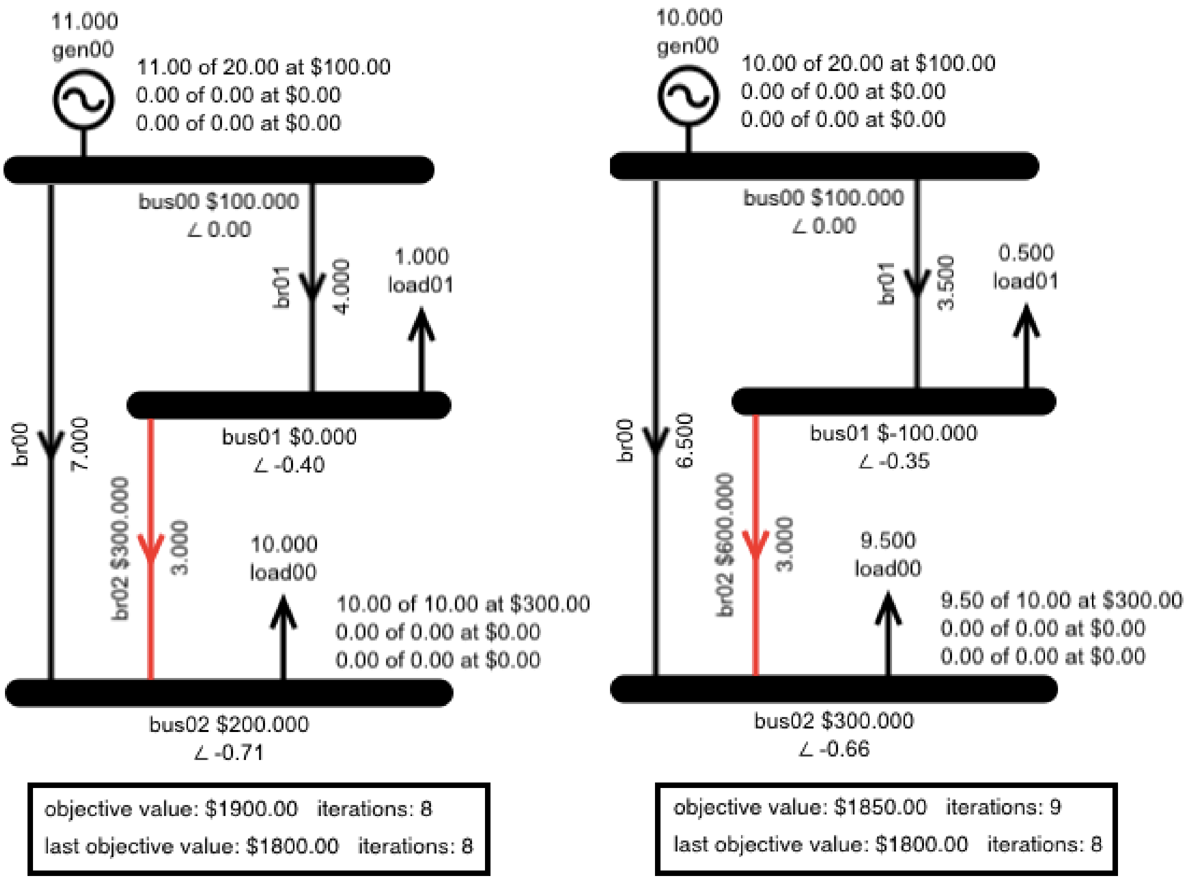


Figure 56: Extra load at bus01 (a) 1MW or (b) 0.5MW result in same $/MW increase in the objective value.

at bus02 to increase in order to keep the binding branch br03 at its maximum. Changing the angle at bus02 allows a corresponding increase in the flow on br00; as observed previously, the total flow across br01 and br02 is exactly the restriction on br00, because overall they both connect bus00 and bus02. The increased flow on br00 allows more of the bid at bus02 to be cleared, improving the objective value (the value of the cleared bids less the cost of the cleared generation), which is now:

$$objective = \$300 \times 10 + \$0 \times 1 - \$100 \times 11 = \$1900 \tag{68}$$

In the original result shown in Figure 55 the objective was $1800, hence this is an improvement of $100 achieved by adding $0 of *load* at bus01. This lines up with the original marginal price at bus01 of −$100.

The objective value and the marginal bus price. Note that the objective value of the current solution and the previous solution are displayed in the top right hand corner of the display. On the iPhone this is outside the display window, so must be accessed by switching on panning and dragging the display across. The objective values for the models we have just solved have been added to the bottom of Figure 56.

We can use this to confirm our calculation in Equation (5.4.1), and also to show that when we added 0.5MW of $0 load at bus01 then the objective improved by $50 (see the result in Figure 56(b). This serves to demonstrate that the marginal bus price is set by the rate of increase in the objective value, not the actual increase. This may not have been clear, because up until now we have always been using a 1MW increase, hence the divisor was always one.

Also up until now the increase in objective value has been discussed in terms of the value of making extra generation available at the bus, but, as this example shows, it is equally valid to talk about the marginal price in terms of the impact of extra load at the bus.

5.4.2 Negative prices cause non-physical losses

When the spring washer effect is active, if the price difference between the load and generation is great enough then eventually prices will go negative, as we saw in Figure 54. As we discussed on page 69 one of the ways of interpreting the negative price is that it is a signal that an increase in load would improve the objective value. In an LMP model where losses are included, the losses are effectively another load on the system. However, unlike the load bids, the losses have no positive contribution to the objective value, only a negative impact as they require more generation to be cleared. However, when there are negative prices this situation changes. The losses on branches that connect to a bus with a negative price can result in an improvement to the objective value, in the same way as extra load at the negative price bus.

We can demonstrate this by building the model shown in Figure 57(a), which is the same as the model we were previously working with, except that:

- We have removed the $0 load that we added at bus01.
- All branches have their max flow set to 10MW, this will make it easier to see the flows when we look at the flow-loss tranches.
- Edit load00 properties to set the bid price to $600/MWh. This is to ensure that we get prices sufficiently negative to cause non-physical losses.
- All other values remain at their default settings.

Figure 57(a) is the control result, in which we note that the losses on br01 are 0.01MW. Now we will cause a spring washer effect by editing the properties of br02 to lower the maximum flow limit to 3MW. This will produce the result shown in Figure 57(b). The negative price at bus01 indicates that additional load at this bus will benefit the objective value. Half the losses on br01 are assigned to bus01, effectively as load. Hence, the result maximises the losses on br01 by breaking the piece-wise linear approximation. This is illustrated by viewing the flow-loss curve for the branches; these are viewed by tapping the results button in the toolbar. This displays the flow-loss curve for the branches, where a selection wheel is used to select which branch to view. The complete list of these results is shown in Figure 58. The only thing keeping the result on the flow loss curve is the fact that increased losses usually result in a worse objective value. When this is no longer the case, there is nothing to

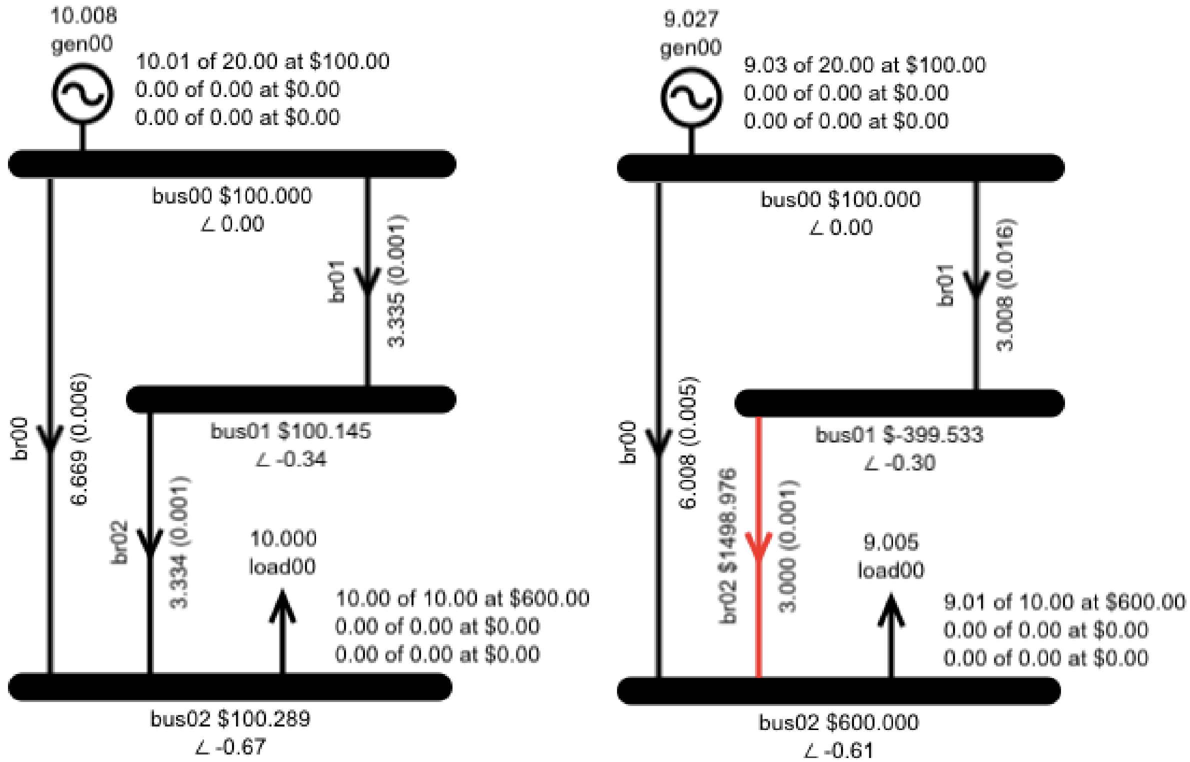


Figure 57: Spring washer with losses (a) Control case with no spring washer effect (b) Spring washer effect increases losses on br01.

prevent the result from over-scheduling the losses, as can be observed for br01. The result for br00 shows a normal result, where the flow-loss point is on the flow-loss curve. The result for br02 shows that it is at its limit, still on the curve.

In the actual electricity market, the presence of non-physical losses is detected by post-processing; given the scheduled flow, the post-processing code can calculate the expected losses and compare this with the loss value from the results. In the event of non-physical losses being detected, the post-processing code will attempt a fix. One such fix is to solve a mixed integer linear programming model that only allows one flow-loss segment to clear for each branch; this is the method employed by the New Zealand electricity market. In the Singapore electricity market, a post-processing algorithm is employed that redefines the flow loss tranches so that only those either side of the scheduled flow remain, then initiates a re-solve. In both cases there are re-checks and subsequent iterations are allowed for, up to a limit.

5.5 Case Study: Hawkes Bay subset of the NZ Power System

Figure 59 shows a portion of the New Zealand transmission system, taken from the System Operator's Substation Location Map [49]. The red lines represent transmission circuits that run at a voltage of 110kV, the orange lines are 220kV. The small squares represent generators, the colour indicates the generation type: blue is hydro, orange is geothermal, white is wind,

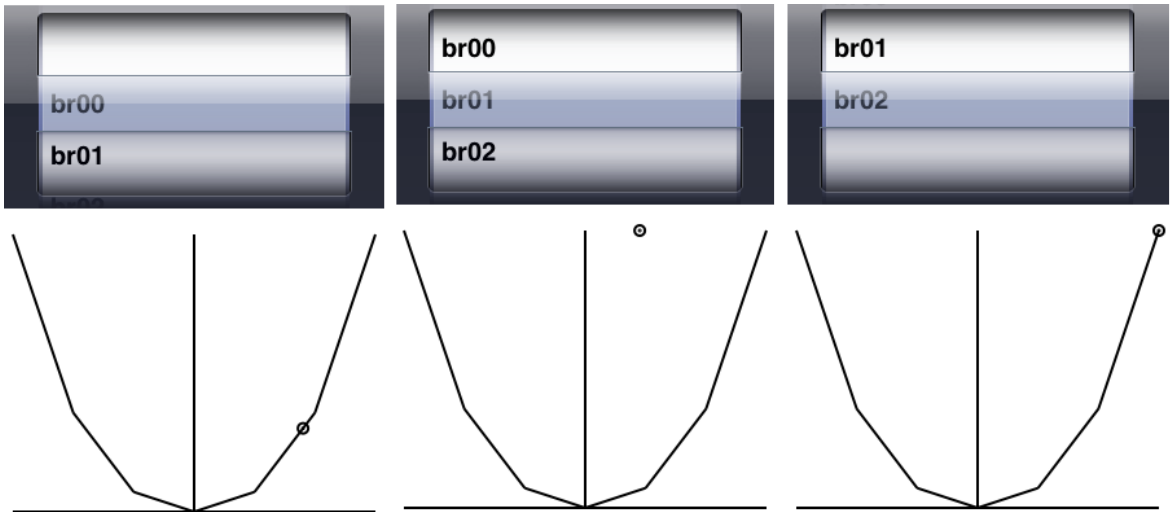


Figure 58: br00 partly loaded, br01 has non-physical losses, br02 is at its limit.

and red is thermal. Thermal generation is either gas, coal or diesel; in this case the only thermal visible on the map is Whirinaki, a diesel generator (which is very expensive to run) built to be run at times when generation is in short supply [76].

From Bunnythorpe the 220kV circuits run south to Wellington and north to Auckland, via Wairakei. At Wairakei there is a 220kV line that travels across to Redclyffe. At Bunnythorpe there are interconnecting transformers (as discussed in Section 3.2.1) that connect the 220kV to the 110kV, and the 110kV circuits run north to Redclyffe. At Redclyffe interconnecting transformers connect the 110kV to the 220kV.

Because the 220kV and the 110kV have a common connection point at Bunnythorpe and another common connection point at Redclyffe, they are effectively in parallel. As discussed in Section 5.3.3, the limit on a branch can also limit the flow on parallel branches. Compared to the 220kV circuits, the 110kV circuits have a higher maximum flow, or *rating*. Hence there is the potential for the lower rated 110kV circuits to restrict the flow on the 220kV. This is prevented by the the Fernhill-Waipawa split [45]; the 110kV circuits traveling north from Bunnythorpe do not have an electrical connection to Waipawa because there are switches open at either Waipawa or Fernhill [23]. The Hawkes Bay has $n - 1$ security (as descibed in Section 3.3) because there are two 220kV interconnecting transformers at Redclyffe.

The Fernhill-Waipawa split allows us to easily model the Hawkes Bay as a subset of the North Island power system, because it means that the only connection between the Hawkes Bay 110kV and the rest of the New Zealand power system is via the interconnecting transformers at Redclyffe. We can build a self-contained model of the Hawkes Bay 110kV where the inflow from these transformers is replaced by a dummy generator. Figure 60 shows the Hawkes Bay portion of the schematic diagram that represents the electricity market model of the New Zealand power system [47]. Figure 61 shows the same section of the system built as a self contained system using the LMP app. [24]

[23] The switches are usually only open at one end, so that the circuit is kept live, this way the System Operator knows that it is available; if the 220kV into the Hawkes Bay are unavailable then the switches can be closed to

Figure 59: Geographical view of the Hawkes Bay portion of the New Zealand power system [49].

Naming of buses and branches. The only feature notable in the building of this model compared to the smaller models we have built is that the buses and branches have been named. The names used for the buses line up with the standard three letter substation abbreviations used by Transpower [48], for example Tuai substation has the abbreviation TUI, Gisborne has the abbreviation GIS. Figure 60 is a Transpower diagram where the bus names consist of the three letter abbreviation and the bus voltage. The names are applied to the components via the properties display for that component. This is available for all of the visible components, i.e., bus, branch, gen, load. To save having to name all the branches the LMP app has the option to set all the branch names automatically, based on the name of the bus at each end, combined with a running count that starts at 1. To run the code that automatically names the branches select the "Auto name branches" option from the Settings display.

---

connect the 110kV circuits.

[24]The Hawkes Bay 110kV is also interesting to model because it includes generation; other portions of the power system could be split off and modeled but they would not include generation. The generation on the Hawkes Bay 110kV is the Waikaremoana power scheme [21] owned by Genesis Energy. The Waikaremoana scheme consists of three generating stations; Tuai (TUI), Kaitawa (KTW) and Piri (PRI). The generation from these stations is injected into the grid at Tuai substation. In the event that the 220kV interconnecting transformers at Redclyffe tripped, it is potentially possible for the Hawkes Bay 110kV to function as a viable electrical island with Waikaremoana as its only power source. If we wanted to model a slightly larger system we could have included the Hawkes Bay 220kV in which case we would also have had the Whirinaki generation available.

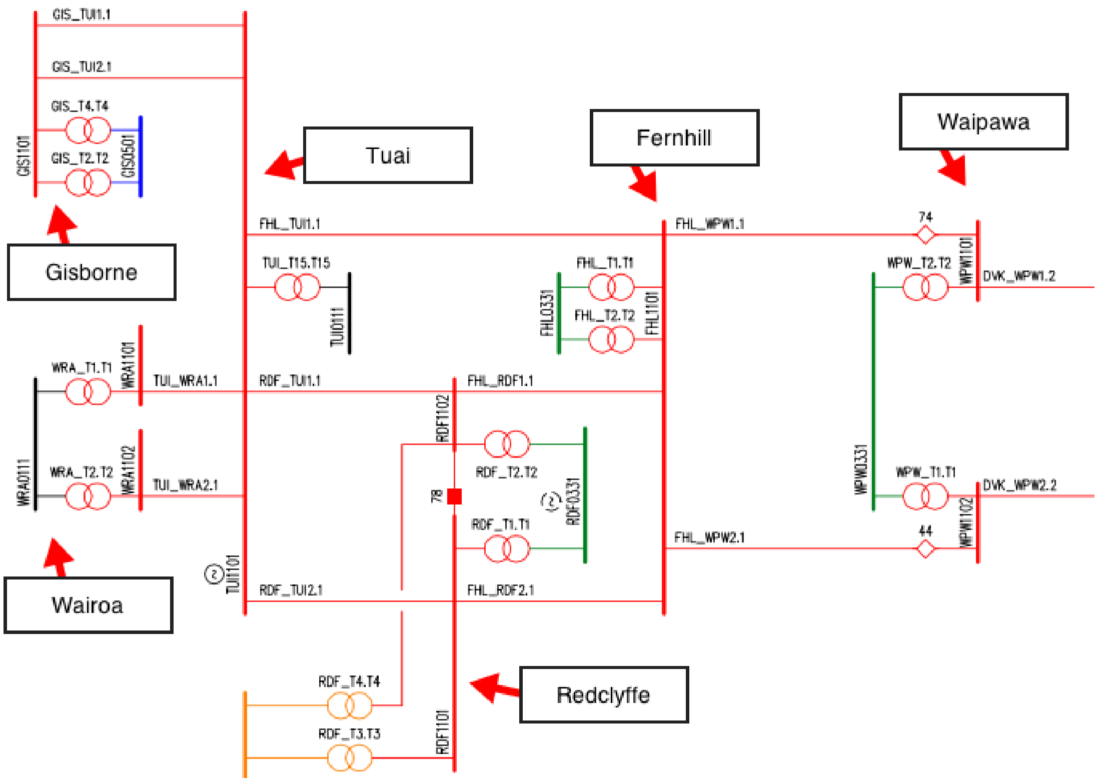


Figure 60: Schematic view of the Hawkes Bay portion of the New Zealand power system [47].

Loading the Hawkes Bay model as a pre-prepared case. After the Hawkes Bay model was built it was saved on the iPhone so that it could be made available as a pre-built model. To load the model, tap on the load/save button . This will take you to the load/save display. From here click on the case name which will take you to the load display, which shows a screen shot of the case. From here clicking on the "Load" button will load the case as the active model.

Viewing all of the Hawkes Bay model on an iPhone display. The Hawkes Bay model is too big for all of it to fit on the iPhone display at once. To view all of the model you will need to either zoom in and out by using the pinch-zoom gestures, or enable the panning feature. Panning involves dragging the drawing area into the display window, however this can result in accidentally dragging components instead. In order to avoid this problem we turned panning off when we first started. If pan is turned off then the pan button is greyed out . Pan can be turned back on by tapping the pan button, which will turn panning on . With panning turned on, the drawing contents can be dragged into position by holding your finger on the screen and moving it as required. When panning is on, components can only be selected for adjustment by tapping them first.

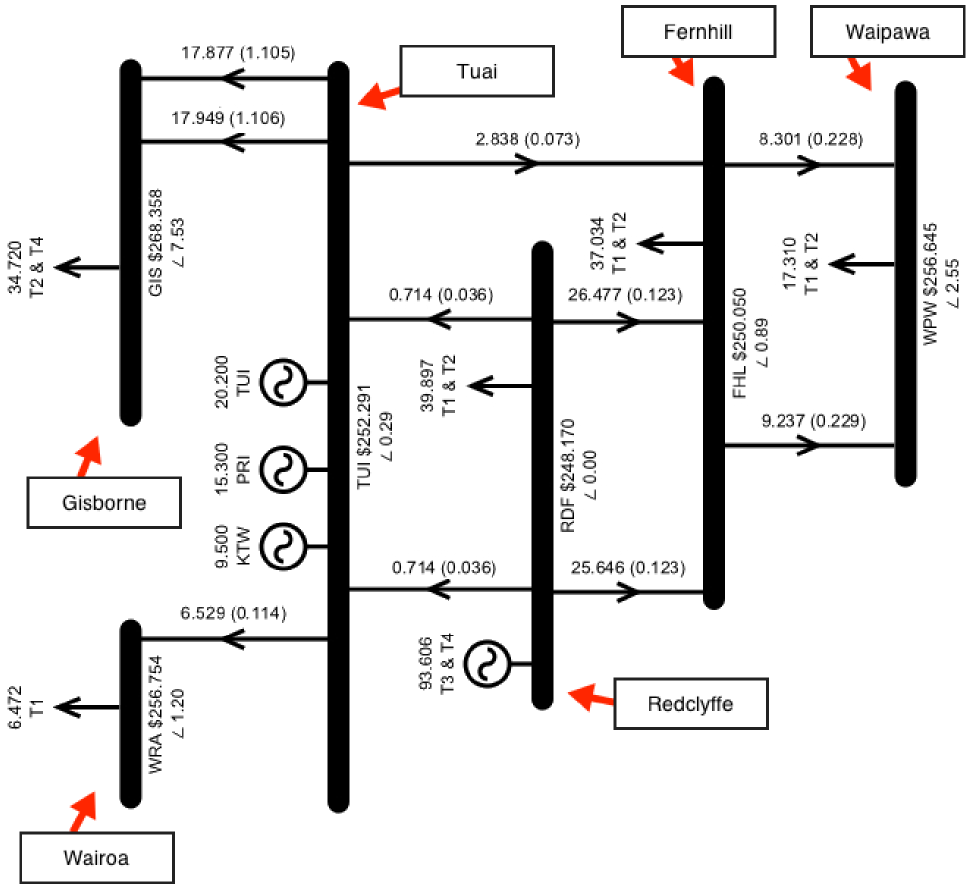


Figure 61: Hawkes Bay portion of the network model implemented in the LMP app.

Solving the Hawkes Bay model using real data. With this model built in the LMP app and the parameters of the circuits entered from the real system, we can then take a schedule from the real market system and enter the time varying parameters into the model in order to try and replicate the solution. The time varying parameters are the generation offers at Tuai and the load at all the other buses. The Redclyffe interconnecting transformers are modelled as a generator with an offer quantity equal to the flow scheduled in the real market and an offer price equal to the Redclyffe bus price from the real schedule [25].

Implementing this using the LMP app produces the model shown in Figure 61. There were initial problems with getting this model to work; some loads were not cleared properly. This LMP model uses bids for the load, whereas the actual electricity market (the SPD software) uses constraints to apply the load. Using constraints requires that there also be variables that allow the load constraints to be relaxed, otherwise a failure to meet the load would cause the LMP model to be infeasible. Using bids is easier because there is no *requirement* to clear the bids, it is only price determining the load result; more of a Lagrangian approach to the problem. Given that this is the case, as the LMP app is capable of working

[25] Input data for the pricing schedules of the New Zealand electricity market is made available by the Electricity Authority on their website [7].

Branch	SPD Flow	App flow
Branch	SPD flow	App flow
FHL-RDF1	26.112	26.477
FHL-RDF2	25.292	25.646
FHL-TUI1	3.45	2.838
FHL-WPW1	9.961	9.237
FHL-WPW2	7.545	8.301
GIS-TUI1	17.727	17.877
GIS-TUI2	17.68	17.949
RDF-TUI1	0.364	0.714
RDF-TUI2	0.364	0.714
TUI-WRA2	6.571	6.529

Table 3: Branch flow results from New Zealand electricity market software (SPD) vs results from LMP app.

properly for smaller models then it seems likely that the costs need to be reconsidered in order to get the Hawkes Bay model to work properly.

Increasing the bid price to \$100k caused the Hawkes Bay system to solve successfully, although only if it were completely separated from the rest of the system, i.e., modelling the inflow as a generator was not working. Comparing the LMP app model with the SPD software, the SPD software uses violation penalties for breaking the load constraint. These penalties have a cost of \$600k. Even though these only set the price if the load constraint is broken, it is these values in the reduced cost equation that are influencing the result. Increasing the bid price to \$600k at all load points in the LMP app caused the system to solve successfully, as shown in Figure 61. This model takes 93 iterations of the simplex algorithm to solve, which takes 17 seconds of elapsed time.

Tables 3 and 4 show the branch flow and bus price results from the LMP app compared with those from the actual electricity market result that was produced by the SPD software. Overall the results are similar in terms of the distribution of prices and the direction and relative levels of the branch flows. The losses are different, possibly due to the fact that the LMP app models the branch losses as being applied 50-50 to the buses at each end of the bus, whereas the New Zealand electricity market model applies the branch losses to the receiving end [26].

5.6 Case Study: Demonstrating the Transmission Pricing Methodology proposal

Transpower is the owner of the New Zealand transmission system. The transmission system is used to transport electricity from the generation to the load. The generators and the purchasers are charged for using the transmission system. The existing methodology for determining who pays how much is complicated and time consuming. The Electricity Authority has proposed a new Transmission Pricing Methodology (TPM) [10], which has been

[26] The New Zealand market originally applied the losses 50-50 to each end. The Singapore market applies them 50-50. The LMP app applies losses 50-50 because it was easier to write the code this way.

BUS	SPD price	App price
FHL	250.268	250.05
GIS	274.1387	268.358
RDF	248.1656	248.17
TUI	249.5007	252.291
WPW	258.7663	256.645
WRA	250.604	256.754

Table 4: Bus price results from New Zealand electricity market software (SPD) vs results from LMP app.

quite controversial [9]. We can use the LMP to demonstrate the basis of the proposal.

The TPM proposal is that the contribution that a purchaser or generator makes towards the cost of a given transmission circuit is the benefit that the purchaser or generator (the payer) receives from the transmission circuit, *as determined by the electricity market SPD software* [27]. The proposed methodology for determining the benefit of a specific transmission circuit is to solve two LMP models; one with the transmission circuit in service, one identical in all ways except that the transmission circuit is removed. The benefit gained at a particular bus due to this transmission circuit is the difference in bus price resulting from removing the circuit from the model. This price difference would determine how much purchasers or generators at that bus would be liable to contribute towards the cost of the transmission circuit. The LMP app can be used to demonstrate this is as follows. Figure 62 shows the price at TUI bus under *base case* conditions, while Figure 63 shows what the price at TUI bus would be if the FHL-TUI-1 transmission circuit was removed. Hence the benefit that the trader at TUI gains from this circuit is $\$252.291 - \$236.105 = \$16.186$

The TPM design leaves a number of questions that are still to be resolved, if indeed the industry accepts the proposal, such as which results would provide the source for the base case, how often would the calculations be updated, would there be processing to remove anomalous results, which software would be used, how would issues such as consequential binding constraints be handled, etc. Note also that the price difference when removing the line need not always be positive. While we could not use the LMP app to answer these questions, we could use it to demonstrate the impact of consequential binding constraints and also how it can be that the benefit is negative.

5.7 Summary

In this section we demonstrated how to use the LMP app to build simple models that explain how and why LMP clears bids and offers, and models that show the impact of the power flow constraint on branch flows, through to building models that explain such things as the need for system splits in the real power system and what causes the results produced by the "spring washer effect". We looked at how the LMP model is presented to the simplex solver and how the results that the simplex solver produces are interpreted. We demonstrated how

[27] Or equivalent software. The Electricity Authority has written their own LMP formulation model which replicates the SPD result [7].

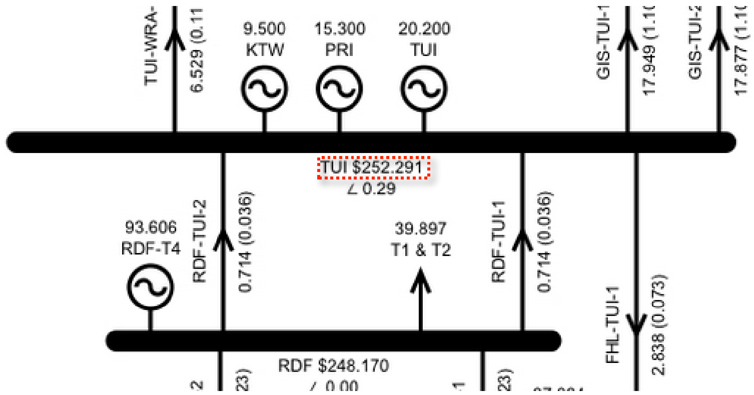


Figure 62: Bus price at TUI $252.291 with all circuits in service.

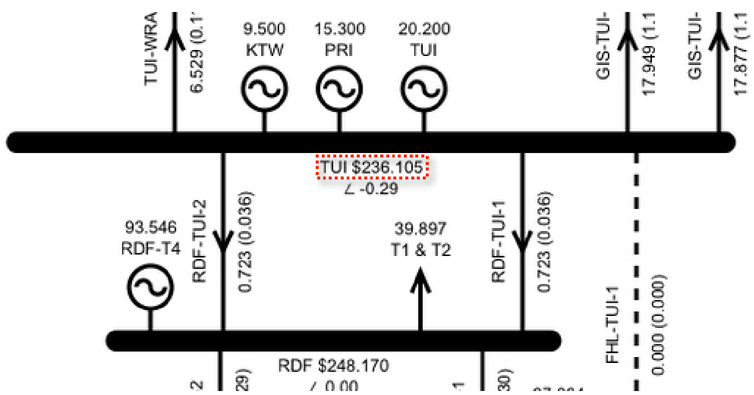


Figure 63: Bus price at TUI $236.105 with all FHL-TUI-1 transmission circuit removed.

the marginal price represents the value of extra generation at the bus and/or the value of extra load. We used the app to show how the "spring washer effect" can result in negative prices that can lead to the failure of the piece-wise linear approximation that is used to model branch losses. We used the LMP app to model a portion of the real power system, where we found that after raising the bid prices to the levels similar to the deficit generation levels used in the actual electricity market model, the results of the LMP app compared reasonably well with those of the actual electricity market. We also used the LMP app to demonstrate how the proposed transmission pricing methodology would determine the benefit that a particular location gains from a given transmission circuit.

6 Conclusion

The goal of this project was to provide a way of explaining the LMP model of a national electricity market in a manner that is quick, visual and adaptable. This was achieved by creating the LMP app which allows an LMP model to be built and solved on an iPhone or iPad. This was combined with an explanation of the details of the LMP formulation, which in turn required a description of the electricity market and the power system which are modelled. To describe how the power system is modelled required a certain amount of electrical theory, explained in terms of concepts that could be understood by someone with only a basic knowledge of electrical circuits.

Following the explanation of the LMP formulation, a sequence of tutorials presented the workings of LMP via a progression of models built using the LMP app. The tutorials started with very simple models that demonstrated how bids and offers clear and how the marginal price is set. The tutorials explained the impact of parallel flows on the electricity market results and also their impact on the real electricity network. The tutorials went as far as building models that explained the spring washer effect and non-physical losses. It was shown that the LMP app can be used to demonstrate all significant features of LMP.

It was also shown how the LMP app can be used to model a portion of the New Zealand power system, and this was used to demonstrate how the proposed transmission pricing methodology could determine the value of transmission circuits. A direct comparison of the LMP app results with a result from the actual New Zealand electricity market system showed differences in flows and prices. These were most likely due to differences in the way that the two systems model branch flows and losses; overall the results were similar in terms of the distribution of prices and the direction and relative levels of the branch flows. In terms of validation of the solution algorithm (the solver), the LMP app includes the facility to export the LP model to the GLPK software. Given the same LMP model as the LMP app, the GLPK solver produces an identical result.

The LMP app includes the constraints essential to demonstrate the concepts of LMP. If there were more time, we could include some further aspects of LMP; specifically, ramp rates, reserves and branch group constraints would have been included if there was more time. Another possible extension would be to update the modelling of branch losses so that the losses are assigned to the receiving bus, as is done in the New Zealand electricity market model; this may help the results to line up more closely.

Building bigger models to find the limitations of the platform is another possibility for the future. Currently the Hawkes Bay model demonstrated in Section 5.5 is the largest model. This takes 96 iterations of the simplex algorithm to solve, which is 17 seconds of elapsed time. This is an iPad sized model which fits on the iPhone by zooming and panning. The software could be adapted to allow models to be built that are bigger than the iPad screen. Possible models that could be attempted are a simplified version of the North Island or South Island system. A simplified North Island system could be used to demonstrate large scale scenarios such as the undesirable trading situation (UTS) that occurred in New Zealand on 26th March 2011 which resulted in $30,000 prices and a spring washer effect that affected most of the top half of the island [6].

Two 90 minute presentations were made to colleagues at Transpower where the LMP app was used to explain the LMP formulation. These were well received, with specific feedback as follows:

- “Thanks for the SPD session... makes a lot of sense on how the cplex thing does what it does”.
- “Today I found the app very useful in helping me understand more about the solver and specifically how the marginal price is determined once all of the offered generation has been cleared, both when all of the bids have cleared and when there are outstanding unfilled bids. It was also good to go over the objective function and the constraints which must be defined to ensure flow in and flow out are balanced”.

The most obvious application of the LMP app is to provide an introduction to LMP. n a more ambitious note and with some more development, another possible use for the LMP app is as a tool for explanation and investigation of proposed changes to LMP. There is ongoing work looking at ways of improving the operation of the electricity market and as often as not these involve the LMP model in some way or other. However, not many people deal with the workings of LMP on a daily basis, hence familiarity with the LMP formulation is something that is not commonly required. When changes are proposed, time is spent explaining them and their impact. Being able to send out case files which illustrated the problem, which could be loaded into the LMP app, would save time, improve understanding and may also allow for people to carry out their own investigations. More widespread understanding of the issues may lead to overall improvement in the design and running of the electricity market.

On a personal level, writing the LMP app software and then using the LMP app to create scenarios that illustrate the LMP formulation has given me the opportunity to (and forced me to) investigate the workings of the LMP formulation in thorough detail. With this project I have also explained what it is that I do. The last 24 years of my working life are wrapped up in this project document and because all of this is necessary background to understanding the LMP formulation, ultimately this is all wrapped up in the LMP app. It is the story of the work that I do and the work that I have done, and if I want to explain to someone how some of it works then now I can pull out my phone and show them.

A Per unit values

The calculation of power flow is complicated by the fact that different parts of the power system run at different voltages. To remove this complication the power flow calculations use per unit values:

$$\text{per unit value} = \frac{\text{actual value}}{\text{base value}} \tag{69}$$

In order to be able to ignore the voltage, we adjust the resistance and reactance values that we use so that they take into account the voltage that they are designed to run at. This is called using *per unit* values and Huang [32] provides a thorough description.

The nominal (or nominated) voltage of a bus is the voltage that it is designed to run at, e.g., the nominal voltage of a 33kV bus is 33kV. The System Operator controls the voltages on the power system so that all voltages are within a pre-defined limit of their nominal voltage. An assumption of the simplified power flow is that all buses are at their nominal voltage.

For the per unit system, each voltage level needs a base resistance and reactance value and a base voltage. All resistances and reactances at this voltage level will be adjusted by dividing them by the base values before they are used, in this way all resistance and reactance values in the system will be relative to their voltage and hence the voltage does not need to be considered in any equations.

For the base voltage we choose the nominal voltage, hence if a voltage is equal to the nominal voltage then its per unit value is 1.0. For a transmission circuit designed to run at 110kV with a resistance of 10 ohms, we want to calculate a per unit resistance. We need to know a base resistance, but we don't quite have enough information. From Equations (1) and (2) we know that $P = fracV^2R$, so given that we know the base voltage is the nominated voltage, if we had a base power value then we could calculate a base resistance. Hence a base power value is defined that applies to the entire system. In the case of the New Zealand electricity market model, the base power is 100MVA, a value that is chosen so that the per unit values end up being not too big and not too small.

B GLPK Comparison

The following is the GLPK model for the model shown in Figure 46.

```
set System;
set Buses;
set Branches;
set Bids;
set Offers;
set BranchSegments;

var busAngle{bus in Buses};
var branchFlow{br in Branches};
var branchFlowPos{br in Branches}>=0;
var branchFlowNeg{br in Branches}>=0;
var branchLoss{br in Branches}>=0;
var segmentFlow{branchSegment in BranchSegments}>=0;
var segmentLoss{branchSegment in BranchSegments}>=0;
var offerCleared{offer in Offers} >= 0;
var bidCleared{bid in Bids} >= 0;

param refBus{sys in System},symbolic;
param name{bus in Buses},symbolic;
param branchSus{br in Branches};
param branchMax{br in Branches};
param brFromBus{br in Branches},symbolic;
param brToBus{br in Branches},symbolic;
param offerMax{offer in Offers};
param offerPrice{offer in Offers};
param offerAtBus{offer in Offers},symbolic;
param bidMax{bid in Bids};
param bidPrice{bid in Bids};
param bidAtBus{bid in Bids},symbolic;
param branch{branchSegment in BranchSegments},symbolic;
param lossFlowRatio{branchSegment in BranchSegments};
param segmentMaxFlow{branchSegment in BranchSegments};

#limits
s.t. maxBranchFlow{br in Branches}:
   branchFlow[br] <= branchMax[br];
s.t. minBranchFlow{br in Branches}:
   branchFlow[br] >= -branchMax[br];
s.t. maxOfferCleared {offer in Offers}:
   offerCleared[offer]  <= offerMax[offer];
s.t. maxBidCleared {bid in Bids}:
   bidCleared[bid]      <= bidMax[bid];

#power flow
s.t. powerFlow{br in Branches}:
   branchFlow[br] - ((busAngle[brFromBus[br]]-busAngle[brToBus[br]]) *
       branchSus[br]) = 0;

#branch flow
s.t. branchFlowTotal{br in Branches}:
   branchFlow[br] - branchFlowPos[br] + branchFlowNeg[br] = 0;

#flow loss
```

```
#segments are always positive, constrain branchPos and branchNeg, which are
    both always positive
#all other branch flow is with branchFlow, which is branchPos - branchNeg
s.t. branchFlowPosIsSumOfSegments{br in Branches}:
   branchFlowPos[br]
   - sum{branchSegment in BranchSegments : branch[branchSegment] = br}
       segmentFlow[branchSegment]
   = 0;
s.t. branchFlowNegIsSumOfSegments{br in Branches}:
   branchFlowNeg[br]
   - sum{branchSegment in BranchSegments : branch[branchSegment] = br}
       segmentFlow[branchSegment]
   <= 0;
s.t. segmentLossFromFlow{branchSegment in BranchSegments}:
   segmentFlow[branchSegment] * lossFlowRatio
[branchSegment]
   - segmentLoss[branchSegment]
   <= 0;
s.t. maxSegmentFlow{branchSegment in BranchSegments}:
   segmentFlow[branchSegment] <= segmentMaxFlow[branchSegment];

#branch loss
s.t. branchLossSumOfLossSegments{br in Branches}:
   branchLoss[br]
   - sum{branchSegment in BranchSegments : branch[branchSegment] = br}
       segmentLoss[branchSegment]
   = 0;

#ref bus
s.t. refBusZeroAngle{bus in Buses, sys in System : refBus[sys] = bus}:
   busAngle[bus] = 0;

#node balance
s.t. nodeBalance{bus in Buses}:
   + sum{br in Branches : brToBus[br] = bus} branchFlow[br]
   - sum{br in Branches : brFromBus[br] = bus} branchFlow[br]
   - sum{br in Branches : brFromBus[br] = bus or brToBus[br] = bus} 0.5*
       branchLoss[br]
   + sum{offer in Offers : offerAtBus[offer] = bus} offerCleared[offer]
   - sum{bid in Bids : bidAtBus[bid] = bus} bidCleared[bid]
   = 0;

maximize Objective:
     sum{bid in Bids} bidCleared[bid]*bidPrice[bid]
     - sum{offer in Offers} offerCleared[offer]*offerPrice[offer];

data;
param :  System : refBus :=
         system   bus00;
set      Buses :=
         bus01    bus00;
param :  Branches : brFromBus  brToBus  branchSus    branchMax :=
         branch00   bus00      bus01    -9.9009901   100.000;
param :  Bids :         bidAtBus bidMax   bidPrice :=
         load00_bid00   bus01    10.000    300.000
         load00_bid01   bus01     0.000      0.000
```

```
        load00_bid02   bus01      0.000       0.000;
param : Offers :       offerAtBus  offerMax  offerPrice :=
        gen00_offer00  bus00       20.000    100.000
        gen00_offer01  bus00        0.000      0.000
        gen00_offer02  bus00        0.000      0.000;
param : BranchSegments :       branch   lossFlowRatio  segmentMaxFlow :=
        branch00_brSegment00  branch00 0.0033333      33.3333333
        branch00_brSegment01  branch00 0.0133333      33.3333333
        branch00_brSegment02  branch00 0.0300000      33.3333333;
end;
```

The GLPK result is as follows. The results in the Activity and Marginal columns can be compared with the results in Figure 46 to confirm that they are the same.

```
Problem:    new_gl_loss_in
Rows:       23
Columns:    18
Non-zeros:  48
Status:     OPTIMAL
Objective:  Objective = 1996.661135 (MAXimum)

   No.   Row name   St   Activity Lower bound Upper bound Marginal
------ ------------ -- ---------- ----------- ----------- --------
 1 maxBranchFlow[br00]
                B       10.0167                       100
 2 minBranchFlow[br00]
                B       10.0167         -100
 3 maxOfferCleared[gen00_offer00]
                B       10.0334                        20
 4 maxOfferCleared[gen00_offer01]
                NU            0                        -0      100
 5 maxOfferCleared[gen00_offer02]
                NU            0                        -0      100
 6 maxBidCleared[load00_bid00]
                NU           10                        10  199.666
 7 maxBidCleared[load00_bid01]
                B             0                        -0
 8 maxBidCleared[load00_bid02]
                B             0                        -0
 9 powerFlow[br00]
                NS            0           -0            =    < eps
10 branchFlowTotal[br00]
                NS            0           -0            = 0.333886
11 branchFlowPosIsSumOfSegments[br00]
                NS            0           -0            = 0.333886
12 branchFlowNegIsSumOfSegments[br00]
                B      -10.0167                        -0
13 segmentLossFromFlow[br00_brSegment00]
                NU            0                        -0  100.167
14 segmentLossFromFlow[br00_brSegment01]
                NU            0                        -0  25.0415
15 segmentLossFromFlow[br00_brSegment02]
                NU            0                        -0  11.1295
16 maxSegmentFlow[br00_brSegment00]
                B       10.0167                   33.3333
17 maxSegmentFlow[br00_brSegment01]
                B             0                   33.3333
```

```
18 maxSegmentFlow[br00_brSegment02]
                B              0                  33.3333
19 branchLossSumOfLossSegments[br00]
                NS             0            -0            = -100.167
20 refBusZeroAngle[bus00,system]
                NS             0            -0            =    < eps
21 nodeBalance[bus00]
                NS             0            -0            =     -100
22 nodeBalance[bus01]
                NS             0            -0            = -100.334
23 Objective    B        1996.66

No. Column name  St   Activity Lower bound Upper bound Marginal
--- ------------ -- ---------- ----------- ----------- --------
  1 busAngle[bus01]
                 B     1.01169
  2 busAngle[bus00]
                 B           0
  3 branchFlow[br00]
                 B     10.0167
  4 branchFlowPos[br00]
                 B     10.0167           0
  5 branchFlowNeg[br00]
                 NL          0           0             -0.333886
  6 branchLoss[br00]
                 B   0.0333886           0
  7 segmentFlow[br00_brSegment00]
                 B     10.0167           0
  8 segmentFlow[br00_brSegment01]
                 B           0           0
  9 segmentFlow[br00_brSegment02]
                 B           0           0
 10 segmentLoss[br00_brSegment00]
                 B   0.0333886           0
 11 segmentLoss[br00_brSegment01]
                 NL          0           0              -75.1254
 12 segmentLoss[br00_brSegment02]
                 NL          0           0              -89.0374
 13 offerCleared[gen00_offer00]
                 B     10.0334           0
 14 offerCleared[gen00_offer01]
                 B           0           0
 15 offerCleared[gen00_offer02]
                 B           0           0
 16 bidCleared[load00_bid00]
                 B          10           0
 17 bidCleared[load00_bid01]
                 NL          0           0              -100.334
 18 bidCleared[load00_bid02]
                 NL          0           0              -100.334

Karush-Kuhn-Tucker optimality conditions:

KKT.PE: max.abs.err. = 1.78e-015 on row 10
        max.rel.err. = 1.78e-015 on row 10
        High quality
```

```
KKT.PB: max.abs.err. = 0.00e+000 on row 0
        max.rel.err. = 0.00e+000 on row 0
        High quality

KKT.DE: max.abs.err. = 2.84e-014 on column 6
        max.rel.err. = 2.84e-014 on column 6
        High quality

KKT.DB: max.abs.err. = 0.00e+000 on row 0
        max.rel.err. = 0.00e+000 on row 0
        High quality

End of output
```

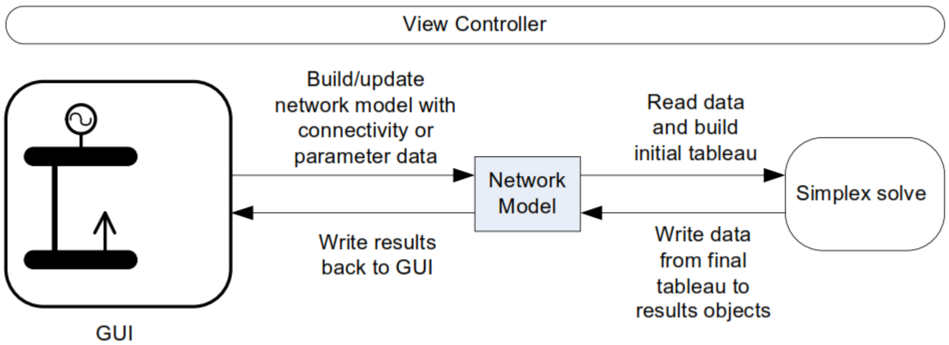


Figure 64: Software components of the LMP app

C Creating the LMP app

Development of the LMP app was initially focused on the iPad, as the size of the display was the original motivation for writing the app. However, once development had progressed to the stage where models could be reliably built and the resulting screen shots emailed off the device, it was found that increasingly it was the iPhone that was being used to run the app. This was due to the convenience of being able to work on the models whenever spare time arose and the device was handy (more likely with an iPhone than an iPad).

Figure 64 shows the functional components of the LMP app software. The LMP components that the user interacts with on the graphical user interface are *views* that are controlled by a *view controller*. While the arrows in Figure 64 show the logical data flow, the views on the Graphical User Interface (GUI) do not know about the Network Model object; data flow is controlled by the View Controller. Similarly when the solve is triggered by a button on the GUI, this button is a view that links to the View Controller and it is the View Controller that initiates the solve. When the solve is complete, it is the View Controller that writes the results back to the GUI.

The GUI is used to add components, connect them together and enter their parameters. Every time the connectivity is changed, i.e., a component added or deleted, or connected or disconnected from another component, the Network Model data is updated. Whenever parameter data is saved, the Network Model data is updated.

The two most complicated parts of the code are the procedure within the View Controller which processes the connectivity changes and the procedure which builds the LMP formulation from the Network Model data.

C.1 The operating system

The LMP app is written to run on iOS, Apple's mobile operating system used by the iPad, iPhone and iPod. The iOS operating system is based on the OS X operating system used by Apple computers, which can trace its origins, via NeXT, back to BSD Unix. At the base of iOS is a BSD Unix kernel.

The app is developed within the Xcode development tool. Within Xcode, the app is coded using the Objective C programming language. Objective C was created in the early 1980s. It was licenced by NeXT in 1988 and from there made its way to Apple. It is an object oriented language based on the C programming language. The C programming language is not object oriented. However any C code can be used within Objective C.

Within Xcode, the displays are designed using a graphical representation called a storyboard. There are separate storyboards for the iPhone and the iPad, but they both link to the same Objective C code. In the Xcode development environment a display that has been built in the iPhone storyboard can be copied to the iPad storyboard, where it will be automatically re-sized.

C.2 Differences between iPad and iPhone displays

Within the code, the only areas that are device specific, i.e., specific to the iPad or iPhone, are the code that creates the tool-bars and the code that presents the data entry displays. The iPad uses popovers for data entry, i.e., the data entry display appears as a separate window that pops up over top of the main window, with the main window still visible in the background. The iPhone does not have popovers; if a display is to be presented to allow data entry then it will replace the main display by means of a segue (an animated "sliding-in"). A navigation bar will allow for the data entry display to be dismissed and the main display return, also be means of a segue.

C.3 Design patterns

The over-arching design pattern is the Model-View-Controller (MVC) design pattern, which is, in Apple's words, "central to a good design for an iOS application" [4]. In this design pattern all objects are either models, views or controllers. The model is what the application does. The main models in the LMP app are the model of the electricity market and the solver model. The view controller looks after how the model is displayed. The views are objects used by the view controller in order to display the model. The views should know as little about the model as possible. It is the view controller that is responsible for manipulating the the views to display the model. This helps to make the views predictable and easy to understand [27].

In the MVC pattern, the visible components of the power system represented by views. As these views are added, i.e., drawn, the view controller passes the data to the network model component which builds a data version of the power system. When the solver button is tapped, the view controller passes the power system data to the solver model, which uses the data to create the variables and constraints that it needs and then uses these to build and solve the LMP formulation. The solver results are passed back to the network model and then the view controller assigns these results to the appropriate views.

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